[redacted]

Progress Report from the Woods Hole Oceanographic Inst.

Dec. 10, 1941

---

1) A second group of 10 Ensigns has received training at Woods Hole and they are now awaiting orders. Their equipment is ready to be shipped. Several officers from the Inshore Patrol of the First Naval District have also been sent to Woods Hole for shorter periods of instruction.

2) About 40 special slide rules have been made up and these greatly facilitate the calculation of the path of the limiting sound ray. In most situations the range at any depth can be known within 5 or 6 minutes after the temperature record is available. A considerable number of these slide rules have been distributed to officers who are interested in the refraction of sound in sea water.

3) A set of 5 charts summarizing the available bathythermograph observations in the Western North Atlantic has been prepared. It is possible that a general distribution of these results should be made independently of the revised report on "Transmission of Sound in Sea Water", which is now ready for final typing.

4) Reports received from various officers who are now using bathythermographs and the auxiliary equipment indicate some changes which will improve the performance of the instrument under operating conditions. Nevertheless, useful observations are now being made in a routine way from destroyers and, considering the difficulties involved, the results of the

present design are most satisfactory. Besides improving the existing model we have begun to build a more rugged, hand operated instrument for use by small patrol craft. It may also be useful for showing up thermal stability near the surface and the resulting poor echo ranging conditions.

5) A temperature-depth recorder for submarines has been built and given preliminary tests.

6) A study of the advantages to be gained by a towed projector (or listening device) as well as a brief analysis of the importance of refraction from the submarine's point of view, both emphasize the possible significance of sound channels. It is believed that sound transmission in such layers should be carefully investigated, if this has not already been done.

7) A bathythermograph has been installed on a ferry which crosses Lake Michigan daily. The Meteorological Department of the University of Chicago is using these observations to study the exchange of heat, watervapor, and momentum. Prof. Rossby and Dr. Church are revising the theory of wind currents in the light of the newer ideas concerning the turbulence in the surface layer and with their most valuable cooperation it is hoped that before long our understanding of the thermal structure in the surface layer will be placed on a sound theoretical basis.

8) Of recent weeks a considerable part of our effort has gone into underwater photography at the request of representatives of the Bureau of Ordnance. However, this work also has an anti-submarine slant in that recent tests show that a light can easily be seen at depths of around 100 feet, and possibly [illegible]

submarine carry a light, she could be seen or photographed from the surface ship through an underwater port.

9) It should perhaps be added that the problem of how best to use the "Atlantis" in the future is a serious one. Moreover, it is not going to be easy to hold together her crew or, for that matter, to keep many of our laboratory technicians and assistants from also enlisting in the Army or Navy.

[redacted] NON-REGISTERED PUBLICATION

Declassified

CALCULATION OF SOUND RAY PATHS
USING THE
REFRACTION SLIDE RULE

May 1943

BUREAU OF SHIPS NAVY DEPARTMENT

WASHINGTON, D.C.

[redacted]

CALCULATION OF SOUND RAY PATHS
USING THE
REFRACTION SLIDE RULE

COLLABORATORS:

Woods Hole Oceanographic Institution.
Division of War Research, University
of California.
National Defense Research Committee.
Bureau of Ships, Navy Department.

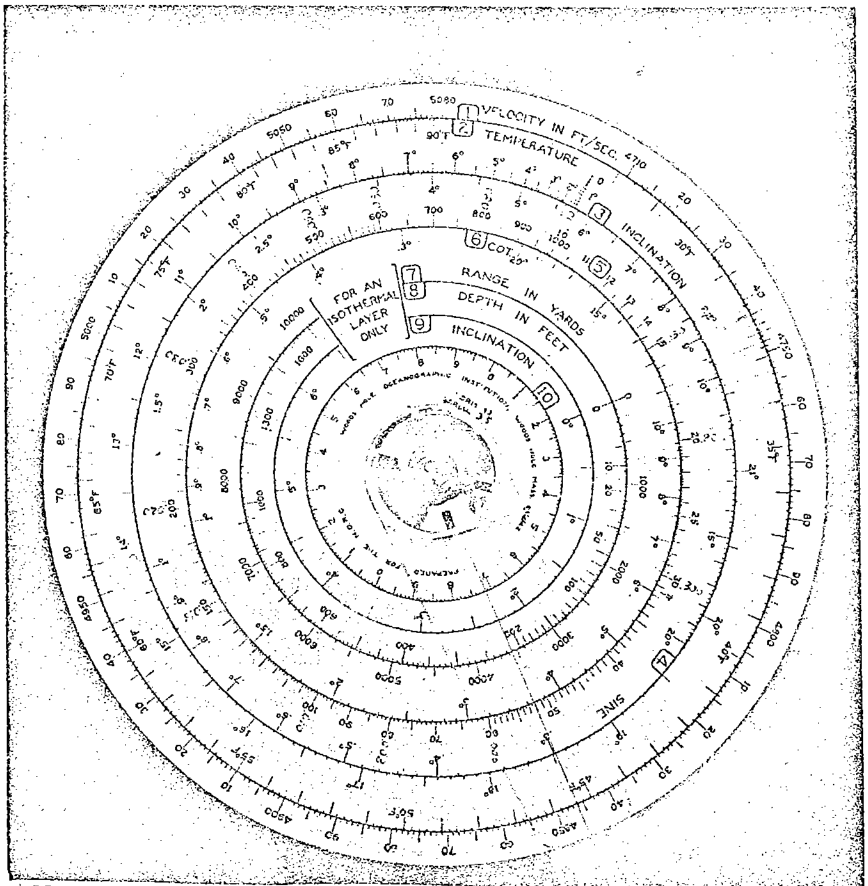
TABLE OF CONTENTS

I INTRODUCTION

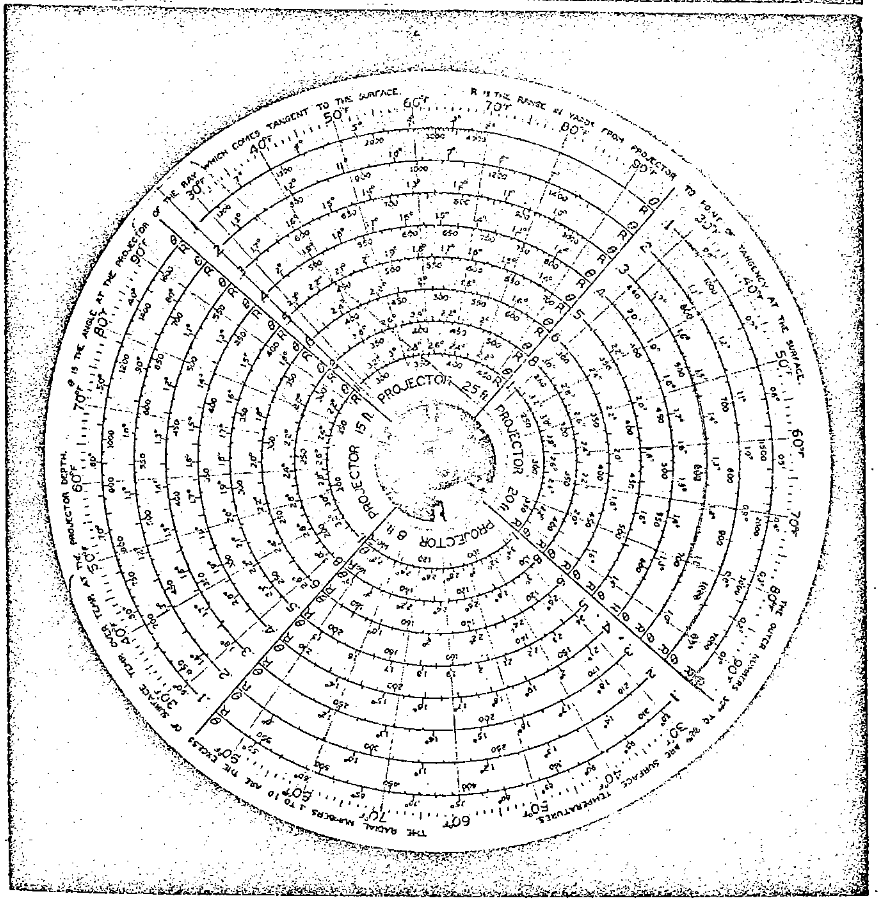
II RAY DIAGRAM CALCULATIONS

III INTENSITY CALCULATIONS

IV PREDICTION OF MAXIMUM ECHO RANGE



1 VELOCITY IN FT/SEC.
2 TEMPERATURE
3 INCLINATION
4 SINE
5
6 COT
7 RANGE IN YARDS
8 DEPTH IN FEET
9 INCLINATION
10
FOR AN ISOTHERMAL LAYER ONLY
WOODS HOLE OCEANOGRAPHIC INSTITUTION
PREPARED FOR THE N.D.R.C.



θ IS THE ANGLE AT THE PROJECTOR OF THE RAY WHICH COMES TANGENT TO THE SURFACE.
R IS THE RANGE IN YARDS FROM PROJECTOR TO POINT OF TANGENCY AT THE SURFACE.
THE OUTER NUMBERS 30° TO 90° ARE SURFACE TEMPERATURES.
THE RADIAL NUMBERS 1 TO 10 ARE THE EXCESS OF SURFACE TEMP. OVER TEMP. AT THE PROJECTOR DEPTH.
PROJECTOR 25 ft.
PROJECTOR 20 ft.
PROJECTOR 8 ft.
PROJECTOR 15 ft.

CALCULATION OF SOUND RAY PATHS
USING THE
REFRACTION SLIDE RULE

1. INTRODUCTION

A. General Statement

The maximum ranges at which a target can be detected at sea with supersonic echo-ranging equipment are very variable and in most instances are not limited by the sensitivity of the equipment itself. Recently, the importance of refraction, or bending of the supersonic beam has been emphasized as a controlling factor in determining maximum echo ranges. Predictions based on the calculated bending of the sound beam have been found extremely valuable in establishing the effective maximum ranges which exist in any area at a particular time.

If the velocity of sound in sea water were everywhere the same, there would be no bending. A beam projected horizontally from a standard projector could be represented by a cone of circular cross section, with a fan shaped section in a vertical plane through the axis as shown in Figure 1. It should be noted that the edge of the shadow zone is not as clear cut as the figure indicates. In practice, a shadow zone is a region from which no useful echo can be obtained.

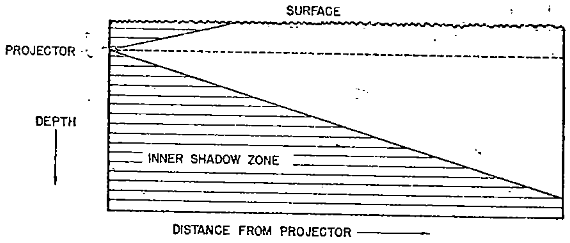


Figure 1

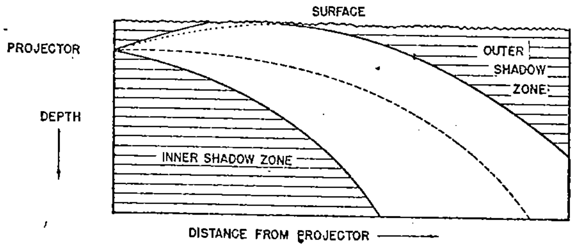


Figure 2

Under most conditions, the sound beam does not travel in a straight line but is bent upward or downward as in Figure 2, split and distorted, owing to changes in sound velocity with depth. The velocity of sound in sea water increases with temperature, with pressure, and with salinity. Ordinarily, salinity differences between the surface and a depth of 600 feet are small and may be neglected because variations of sound velocity with salinity are also small. The increase of sound velocity due to pressure amounts to eleven feet per second at a depth of 600 feet. That is, the portion of the wave front at 600 feet travels eleven feet per second faster than the portion of the wave front at the surface, if the temperature of the water is the same at both depths. A temperature difference of only three degrees Fahrenheit will affect the velocity of sound as much as a pressure difference equal to 600 feet of water. As temperature differences of ten or more degrees in a 600 foot water column are common, and as these differences vary from place to place while the pressure difference is the same all over the world, it is clear that knowledge of the distribution of temperature with depth is essential for the calculation of echo ranges. This information can be obtained at any time from a moving vessel by the use of the bathythermograph, an instrument which gives, on a small glass slide, a record of temperature against depth.

For a quick, but approximate, determination of sound ranging conditions, simple tables have been drawn up. These tables are shown and their use described in "Instruction Manual for Bathythermograph Observers, - Part II, Prediction of Sound Ranges from Bathythermograph Observations", published by the Bureau of Ships.

For some purposes, however, an exact knowledge of the refraction pattern is desirable. When the temperature distribution is complicated, or when information is wanted concerning the variation of maximum echo range with target depth, the simple methods no longer suffice. To simplify the more extended calculations the Refraction Slide rule, the use of which is described in the present Manual, has been developed.

The refraction slide rule can be used to calculate the maximum echo range obtainable from a target at any depth, in any type of water, provided the variation of temperature with depth is known from a bathythermograph observation. Frequently it is sufficient to calculate the path of a single limiting ray which bounds the shadow zone and is horizontal at the depth of maximum velocity. In some cases however, a ray diagram must be constructed, showing the deflections of various rays in the sonic beam. These diagrams can be produced in 2 to 15 minutes, depending on the complexity of the water and the skill of the operator.

If the target is in the beam, the intensity of sonic energy reaching it will frequently depend primarily on the range, regardless of the type of refraction which the beam undergoes in transit. However, in some cases the intensity in part of the beam is reduced by refraction far below the value appropriate to the distance, and this decrease must be taken into account. The slide rule can be used, both to calculate the shape of the ray diagram and to calculate the relative intensity in any part of the ray diagram. With even a little experience in working with ray diagrams, it is quite easy to know in advance which cases are worth the additional time required for intensity calculations.

Thus, the slide rule can be used first to calculate ray paths for the construction of a ray diagram or the determination of the boundaries of the shadow zone, and second, for calculation of relative intensities along any ray in cases where there are abnormal variations of intensity with range. On the basis of these calculations the maximum echo range can be predicted with reasonable accuracy for any type of water conditions.

In depths less than 100 fathoms, where the bottom is smooth and hard and the refraction is downward, extension of the range by reflections from bottom must sometimes be taken into account.

B. Principles of Refraction Calculations

The refraction of sound rays depends on the variation of the velocity of sound from point to point. The treatment of refraction in the present report is based on the following assumptions about sea water, justified in the light of experience.

1. The velocity of sound in the open ocean depends only on temperature and pressure, changes in salinity being negligible.

2. The water is stratified horizontally, which means that a bathythermograph lowering defines the temperature-depth relationnat all points within echo-ranging distance of the point of lowering.

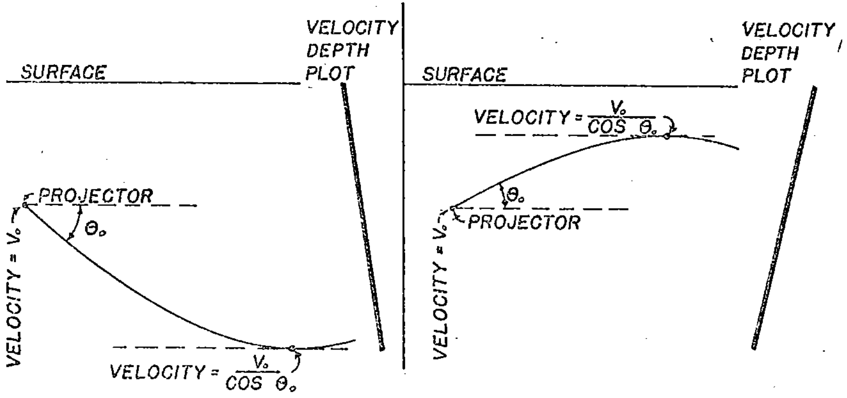
3. The temperature-depth curve on the bathythermograph slide may be approximated by a series of straight lines, which means that the water column may be considered made up of a series of layers, each having a constant thermal gradient.

Any ray passing through a horizontally-stratified medium remains in a single vertical plane. The inclination to the horizontal for any point on the ray is given by Snell's Law . Let V_0, V_1, V_2, etc. be the velocity of sound at points along the ray, and let Θ_0, Θ_1, Θ_2, etc. be the inclination of the ray at the corresponding points. Then Snell's Law, which is the entire basis of ray diagram construction, states that

$$\cos \Theta_0/V_0 = \cos \Theta_1/V_1 = \cos \Theta_2/V_2 = \dots \qquad (1)$$

The following deductions can be made from Snell's Law:

1. Suppose that the velocity of sound increases steadily with depth, as shown by the solid line on the right hand side of Figure 3A, where the velocity (horizontal scale) is plotted against depth (vertical scale). To the left in Figure 3A is shown a typical ray in such a case, where the depth scale is the same as on the right-hand side, but the horizontal scale now represents distance from the projector. This combination of two graphs together is a very useful way of indicating the effect of a particular velocity distribution on the ray diagram. In Figure 3A, V_0 and Θ_0 represent the velocity and inclination of a ray at the projector. Let V_1 and Θ_1 relate to any other point on the same ray. Equation 1 shows that as V_1 increases, Θ_1 becomes smaller, becoming zero when $V_1 = V_0/\cos\Theta_0$, and this particular ray can never reach a level where the velocity is greater than this value. A ray leaving the projector with a greater initial inclination can obviously reach a level of greater velocity. When the velocity decreases with depth, the corresponding path of a ray through its maximum level is shown in Figure 3B.



A. B.

Figure 3

2. Suppose that with increasing depth in the water the velocity increases to a maximum V_m and then decreases, as shown in Figure 4. All rays for which the initial angle Θ_o for the ray at the projector is so small that $V_o/\cos\Theta_o$ is less than V_m will recurve and return to projector level.

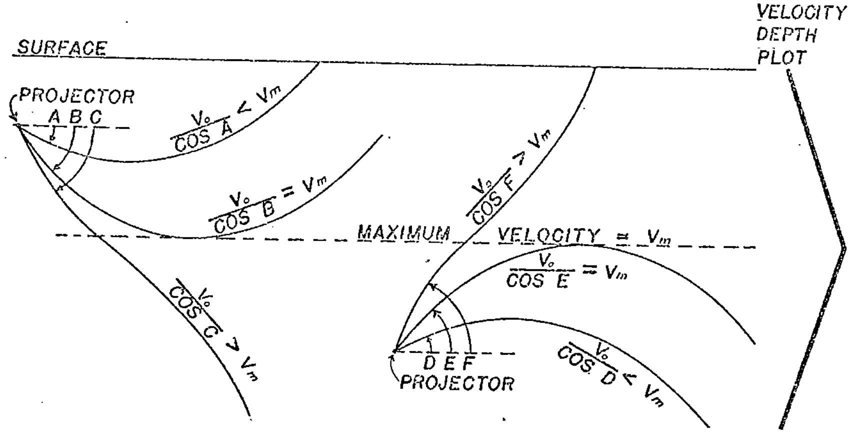
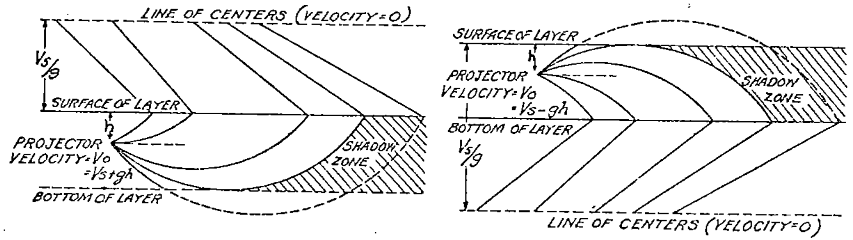


Figure 4

On the other hand, all rays with initial angles, Θ_o, sufficiently large to make $V_o/\cos\Theta_o$ greater than V_m, will penetrate beyond the level of maximum velocity and will not return to projector level.

3. In a horizontal layer in which the temperature-depth graph is a straight line, all rays are circular arcs having centers on a single horizontal plane, as shown in Figure 5. If V_s is the velocity of sound at the surface of the layer, and if the velocity changes with depth at a rate g, the velocity at any depth in the layer is $V = V_s + gh$. The distance from the surface of the layer to the plane of centers is V_s/g. It may assist the memory to note that the plane of centers is the plane at which the velocity would be zero if the layer extended that far and maintained a constant rate of change of velocity.



CASE A. REFRACTION WITH A POSITIVE VELOCITY-DEPTH GRADIENT

CASE B. REFRACTION WITH A NEGATIVE VELOCITY-DEPTH GRADIENT

Figure 5

4. If a ray recurves and thus passes a given depth twice, Equation 1 shows that its inclination on the two passages will be equal in magnitude. For example, if one of the rays in Figure 5 leaves the projector level, with an inclination of +3°, and returns to projector level, its inclination will then be -3°.

5. If the variation of velocity with depth is such that a given ray encounters the same velocity at two different depths, it is seen from Equation 1 that the ray will have the same inclination at both depths.

C. Principles of Intensity Calculations

The method of calculating intensity given in this report assumes that no sonic energy is lost by scattering or by absorption in the water. The loss in intensity is considered to be due simply to the geometrical spreading of the beam. With this assumption, the total energy between any two rays is the same at all distances from the projector; since the intensity is equal to the energy crossing an area one foot square placed perpendicular to the beam, this quantity decreases as the distance between any two rays increases.

When the velocity is constant throughout the water, all rays are straight and the perpendicular distance between any two rays is directly proportional to the distance. If account is taken of the lateral spreading of the beam in addition to the spreading in the vertical plane shown in the simple ray diagram, the familiar inverse square law results; that is, the intensity at any point is inversely proportional to the square of the distance from the projector. Where variations in velocity bend the rays strongly, it is evident from inspection of the ray diagram that there are places where the effect of refraction is to spread the rays apart, decreasing the intensity below the value which would otherwise exist at the same distance from the projector. This is particularly true when a ray reverses its direction of curvature at a point where the inclination is slight. On the other hand, in some cases the rays converge at some points, leading to large increases in the intensity.

Figure 6 contains ray diagrams for two types of refraction. In each ray diagram the angles between successive rays at the projector are all equal, so we may consider that equal amounts of energy go imto each of the spaces between rays. It is obvious that at point A the effect of refraction has been to spread the energy over a wider area, decreasing the intensity, while at point B the effect is the opposite.

All intensities calculated in this report are relative intensities, the intensity at any point being given in decibels (see Page 13) below the intensity at a distance of one yard from the projector, considering the projector to act as a point source of sound.

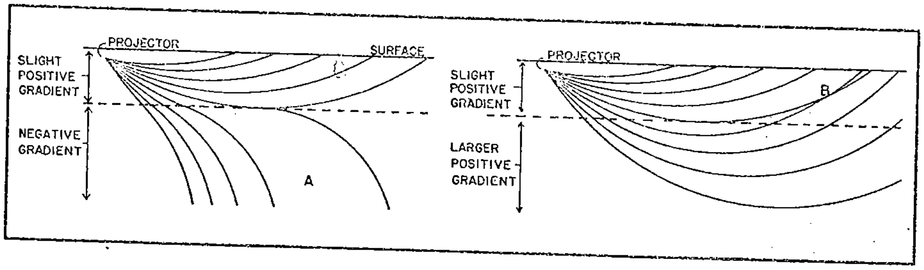


Figure 6

D. Description of the Slide Rule

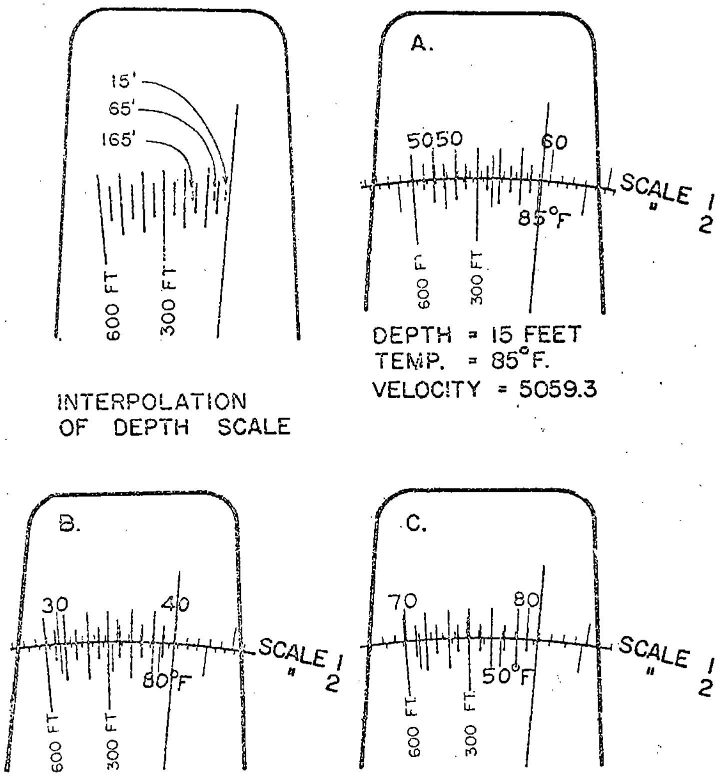
The front side of the rule (see Frontispiece) contains ten circular scales and two index arms. The longer arm contains a long index mark and a scale of depth extending from zero to 600 feet. If the bearings of the rule are in proper adjustment, rotation of the longer arm causes the shorter one to move with it so that the angle between the two remains constant. Rotation of the shorter arm does not affect the position of the longer one.

The scales of the rule numbered from the outside to the center, are as follows:

Scale 1. Velocity of sound in feet per second for sea water. The graduations on this scale are spaced so that the angle included between any two velocity settings, V_0 and V_1, is proportional to $\log V_0 - \log V_1$.

Scale 2. Temperature of the sea water in degrees Fahrenheit. On Scale 2, opposite each velocity on Scale 1, is the temperature at which, in sea water of standard salinity (35 parts in 1000) at atmospheric pressure, sound will have that velocity. To find the velocity at any temperature and depth the auxiliary depth scale of the long arm is used. Set this arm so the given depth coincides with the given temperature on Scale 2, and read the velocity on Scale 1 under the main index line. Illustrations of typical settings appear in Figure 7.

Scale 3. The inclination of a sound ray relative to the horizontal. The graduations of this scale are spaced so the angle included between two settings θ_0 and θ_1, is proportional to $\log \cos \theta_0 - \log \cos \theta_1$, the constant of proportionality being the same as that used for Scale 1. It is used in conjunction with Scales 1 or 2 to solve Snell's Law (Equation 1) in the logarithmic form:



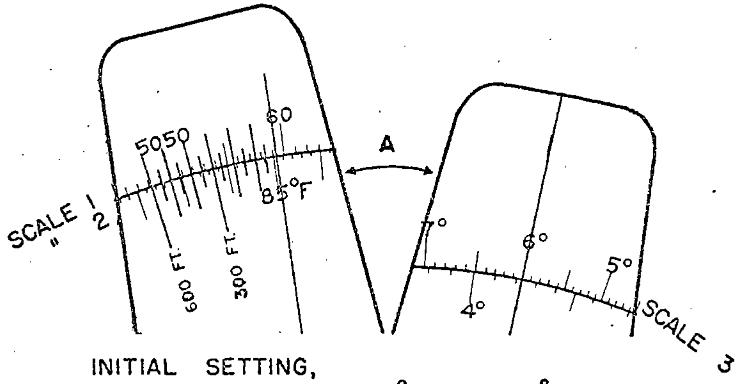
INTERPOLATION
OF DEPTH SCALE

A.
DEPTH = 15 FEET
TEMP. = 85°F.
VELOCITY = 5059.3

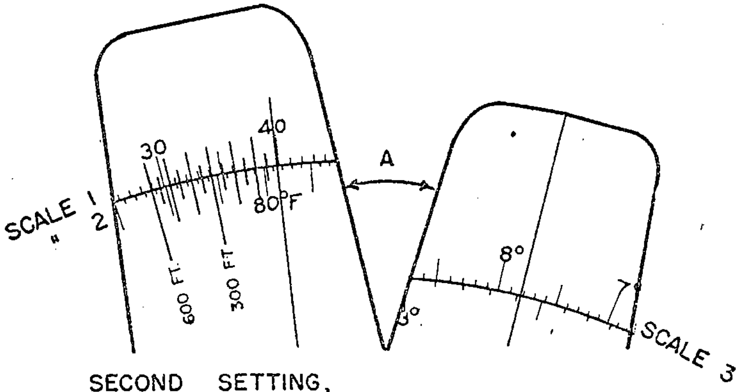
B.
DEPTH = 65 FEET
TEMP. = 80°F.
VELOCITY = 5039.9

C.
DEPTH = 165 FEET
TEMP. = 50°F.
VELOCITY = 4880.9

Figure 7



INITIAL SETTING,
$h_o = 15$ FT., $T_o = 85°$ F., $\theta_o = 6.0°$



SECOND SETTING,
$h_1 = 65$ FT., $T_1 = 80°$ F., $\theta_1 = 7.82°$

NOTE THAT CHARACTERISTIC ANGLE (A) DOES NOT CHANGE.

Figure 8

$$\log \cos \theta_1 = (\log \cos \theta_0 - \log V_0) + \log V_1$$

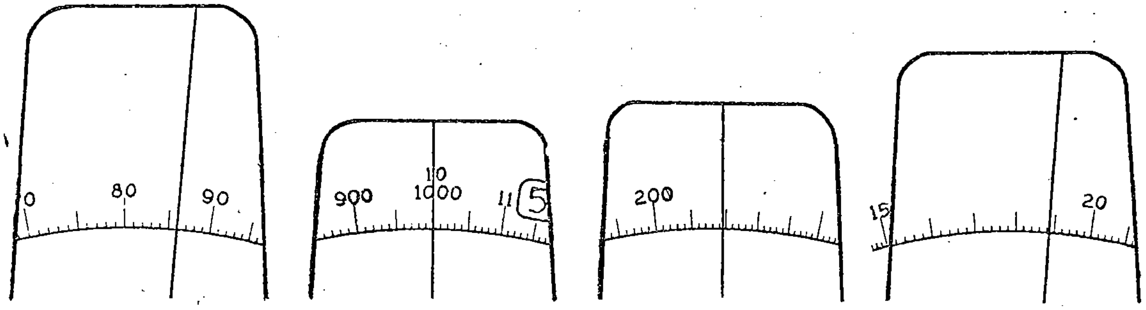
To trace a ray which has the inclination θ_0 at a depth h_0 and a temperature T_0, set h_0 on the long arm to coincide with T_0 on Scale 2 and set the index of the short arm on θ_0 on Scale 3. The angle between the two arms now represents log cos θ_0 - log V_0. This angle between the arms is characteristic for the given ray and should remain unchanged until all inclinations on that particular ray have been determined. Thus, the initial setting in calculations for a given ray consists of arranging the scales to show (1) a depth, (2) a temperature (or a velocity which replaces both depth and temperature), (3) an inclination. To find the inclination of this ray at any other point where the temperature and depth are T_1 and h_1, set h_1 on the long arm in coincidence with T_1 on Scale 2 and read θ_1 under the short index on Scale 3. Figure 8 illustrates this calculation.

One feature of Scale 3 which may require explanation is the method of graduation for inclinations between zero and 2°. By reference to Figure 10B, it is seen that the curved line connecting zero with 2° is divided into ten parts, corresponding to intervals of 0.2°, by a set of parallel lines. The settings of the index are made on the intersections of these lines with the curve.

Scale 4. The sine scale is graduated so that the angle between any two settings, θ_0 and θ_1, is proportional to log sin θ_0 - log sin θ_1. It is related to Scale 5 in such a way that opposite each angle on Scale 4 is the sine of that angle (except for decimal place) on Scale 5. If the user has difficulty keeping the decimal point straight, he may write in the sines of a few representative angles in black lacquer in the space between these two scales, as seen in Figure 20. The sine scale is used only in calculations of intensity. From Figure 10A, it may be seen that sin 5.02° = .0875.

Scale 5. <u>The logarithm scale</u> is a standard two-cycle logarithm scale graduated from 10 to 1000. It is used for all operations of multiplication and division and may be used in conjunction with Scales 4 and 6. In general, the decimal place cannot be determined from the rule.

To obtain the product ab, (1) set the long index on a and the short one on 10, (2) move the long arm until the short index is at b, and (3) read the product under the long index, as in Figure 9, which illustrates the multiplication of 219 by 0.86.



INITIAL SETTING
LONG ARM .86

SHORT ARM 10

SHORT ARM 219

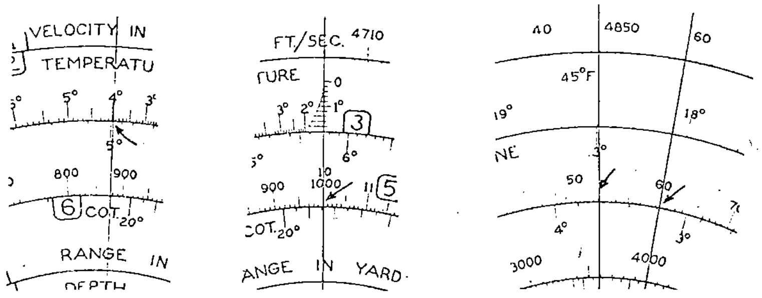
SECOND SETTING
LONG ARM 188 (ANS.)

MULTIPLICATION: .86 X 219=188

Figure 9

Figure 10 illustrates the multiplication 60 x sin 5.02° = 5.26.

To obtain the quotient a/b, (1) set the long index on a and the short one on b, (2) move the long arm until the short index is at 10, and (3) read the quotient under the long index, as in Figure 11, which illustrates the division of 5.88 by 6.82.



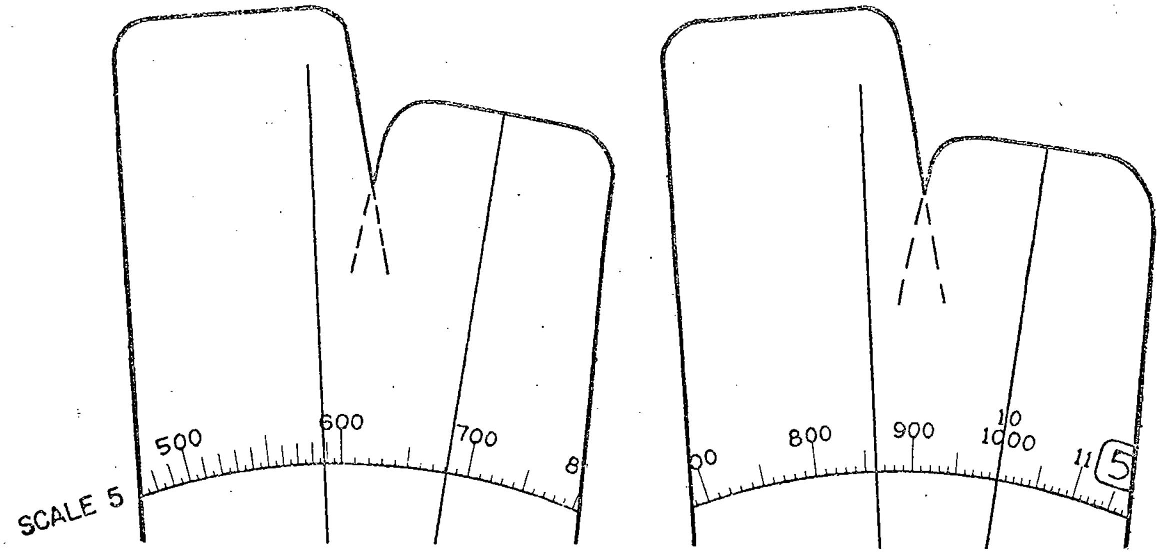
INITIAL SETTING
A. LONG

B. SHORT

FINAL SETTING
C. LONG D. SHORT

60 X SIN 5.02° = 5.26

Figure 10

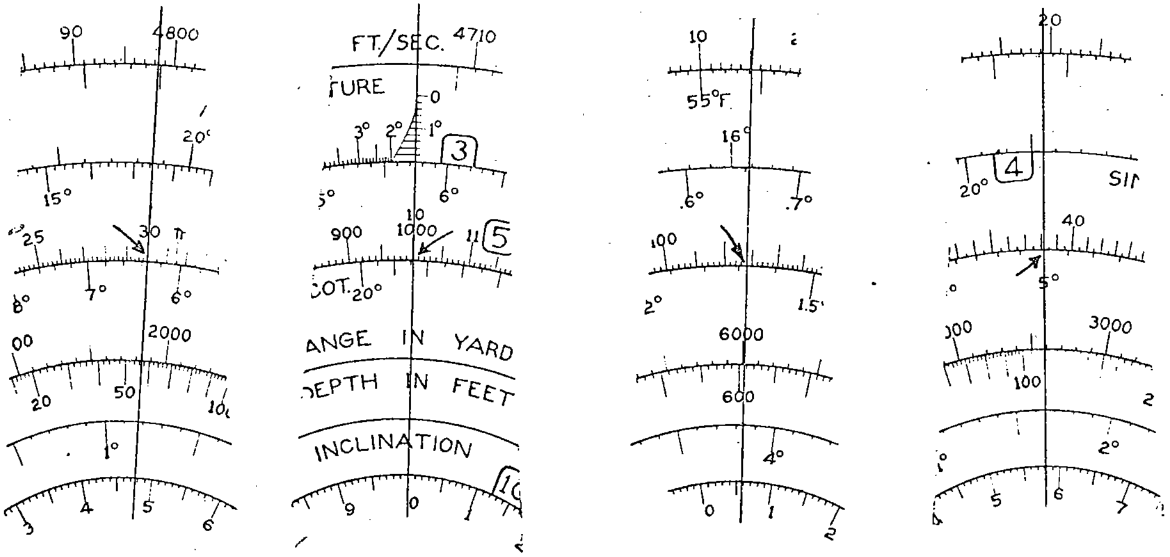


INITIAL SETTING
SHORT ARM 6.82
LONG ARM 5.88

SECOND SETTING
SHORT ARM 10
LONG ARM .86 (ANS.)

DIVISION: 5.88/6.82 = 0.86

Figure 11



A. LONG
B. SHORT
FIRST SETTING
C. LONG
D. SHORT
SECOND SETTING

$\Delta h = 30$ FT., $\tilde{\theta} = 5^\circ$, $\Delta X = 114$ YDS.

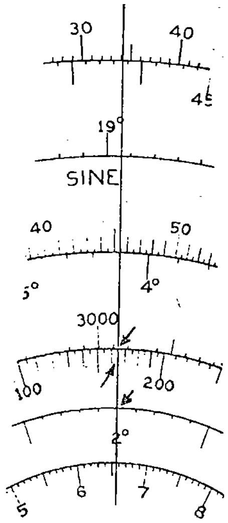
Figure 12

Scale 6. The cotangent scale is used to compute the horizontal travel X of a ray in a layer when the thickness of the layer, h, and the angles of entrance and emergence, θ_0 and θ_1 are known.

The equation involved is

$$X = h \cot\left[(\theta_0 + \theta_1)/2\right] = h \cot \bar{\theta} \quad . \ . \ . \ . \ (2)$$

In the actual use of this scale: (1), on Scale 5, set the long index on h in feet and the short index on 10; (2), move the long arm until the short index coincides with the given mean angle, $\bar{\theta}$, on Scale 6; and (3), read the range X in yards under the long index on Scale 5. This scale has been graduated according to (log cot θ)/3 in order that the range will come out in yards if the depth is expressed in feet. For example, for $h = 30$ feet, $\theta_0 = 6°$, $\theta_1 = 4°$, $(\theta_0 + \theta_1)/2 = \bar{\theta} = 5°$ one obtains $X = 114$ yards, as seen from Figure 12.



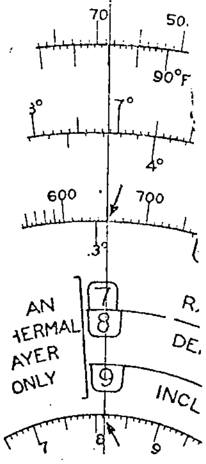
ISOTHERMAL LAYER
θ = 2°, h = 165 FT., X = 3120 YDS.

Figure 13

Scales 7, 8 and 9. These scales are used for isothermal layers only. A single setting of an index line across these three scales gives values of X, h, and θ. X and h are the horizontal and vertical dimensions of a segment of a ray having one end horizontal, and θ is the inclination of the other end of the segment. For instance, a ray with an initial inclination of 2° will travel 3120 yards laterally and 165 feet vertically in isothermal water before becoming horizontal, as seen from Figure 13.

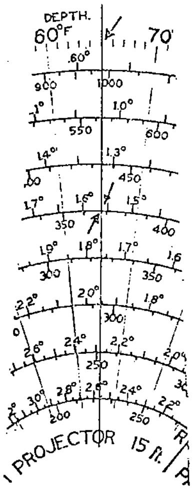
Scale 10. The decibel scale is graduated from zero to 10 in each semicircle, and is used to convert an intensity ratio into decibels. By definition 10 log I_1/I_2 represents the difference in level in decibels between sounds of intensities I_1 and I_2. The ratio of intensities is set on Scale 5 and the corresponding value in decibels is read under the same index on Scale 10. For example when 6.5 is set on Scale 5, one reads 8.1 on Scale 10, which is 10 times the logarithm of 6.5. If the intensity ratio is 650,000 it is necessary to note that the characteristic of the logarithm would be 5, which must be multiplied by 10 to convert it to decibels, and added to the value 8.1 obtained from the slide rule. Thus, an intensity ratio of 650,000 corresponds to 58.1 decibels, as seen from Figure 14. This result may be abbreviated as 58.1 db.

Figure 14



INTENSITY RATIO
650,000 OR 58.1 DECIBELS

Figure 15A



CALCULATIONS WITH REVERSE
SIDE OF RULE.

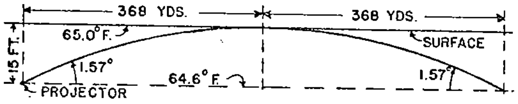


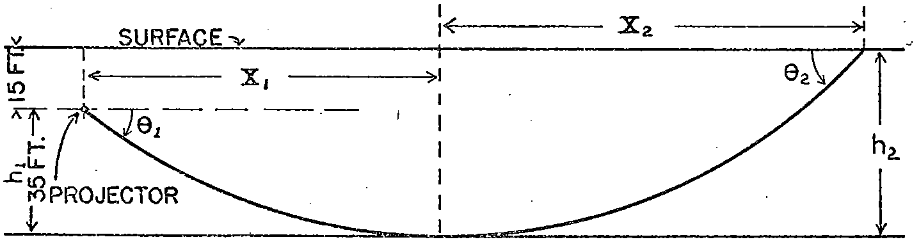
Figure 15B

On the reverse side of the rule (see Frontispiece) are scales used to plot the course of rays from projector depth to surface in cases where there is downward refraction. Each quadrant contains a set of scales for a given projector depth. Suppose the projector depth is 15 feet, the surface temperature is 65°F, and the temperature at the projector is 64.6°F. By setting the index at 65°F in the proper quadrant and reading under it on the 0.4°F scale, we find $\Theta = 1.57°$ and R = 368 yards. These settings and the relationships of these quantities are shown in Figures 15A and B. Further calculation on this ray starts at the end of the segment drawn in Figure 15B where the downward inclination is seen to be 1.57°.

II. RAY DIAGRAM CALCULATIONS

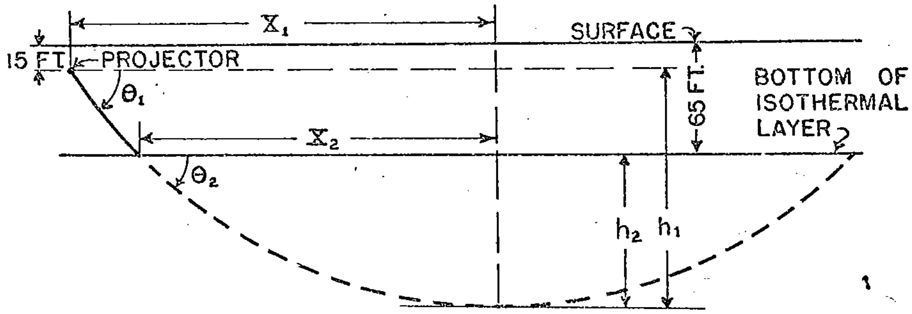
A. Examples on Isothermal Layers.

Examples 1 and 2 below show the use of Scales 7, 8 and 9, the special scales for use on isothermal layers of water. Scales 7 and 8 indicate the horizontal and vertical dimensions of a segment of a ray, one end of which must be horizontal, hence all calculations made with these scales involve breaking all rays up into segments each having one end horizontal.



CALCULATION FOR A RAY WHICH REMAINS ENTIRELY WITHIN AN ISOTHERMAL LAYER. RANGE = $X_1 + X_2$

Figure 16



CALCULATION FOR A RAY WHICH EMERGES FROM THE BOTTOM OF AN ISOTHERMAL LAYER. RANGE = $X_1 - X_2$

Figure 17

All possible situations are included in the two cases illustrated in Figures 16 and 17. In both diagrams, the vertex or horizontal point on the ray represents the common end of the two segments from which the actual ray is built up. A single setting of the index permits the values X_1, h_1, and θ_1, to be read from Scales 7, 8 and 9, while a second setting gives X_2, h_2, and θ_2. From Figure 16, it is clear that the required range is the sum of X_1 and X_2, while in Figure 17 it is seen that the required range is the difference $X_1 - X_2$.

<u>Example 1.</u> Find the maximum range on a surface target in an isothermal layer 50 feet deep which overlies a strong negative gradient. Projector depth is 15 feet.

<u>Solution:</u> The rays bend upward in the isothermal layer, while those which reach the lower layer are bent downward and never return to the surface. Hence the longest range on a surface target is given by the ray which just grazes the bottom of the isothermal layer. Referring to Figure 16, it is seen that h_1, is 35 feet and h_2 is 50 feet. By setting an index on 35 feet on Scale 8, one reads X_1 as 1440 on Scale 7, and by setting at 50 on Scale 8, the value X_2 = 1720 yards is obtained. The maximum range is then the sum of these or 3160 yards. If the angles θ_1 and θ_2 are required, they may be read from the same settings as 0.92° and 1.10° respectively.

<u>Example 2.</u> Find the horizontal partial range in an isothermal layer 65 feet thick for the ray leaving the projector at an angle of 2° below the horizontal. Projector depth is 15 feet.

<u>Solution:</u> Referring to Figure 17 and setting the index of the slide rule at 2° on Scale 9, we obtain X_1 = 3130 yards and h_1 = 165 feet. Since the projector is 50 feet above the bottom of the given isothermal layer, we obtain h_2 = 165 feet - 50 = 115 feet. By setting the index on 115 feet on Scale 8, the values X_2 = 2610 yards and θ_2 = 1.68° may be read on Scales 7 and 9.

The partial range for the given ray in this layer is the difference $X_1 - X_2$ = 520 yards, and the inclination at the bottom of the layer is 1.68°.

It should be recalled that problems dealing with isothermal layers, such as the two examples just given, may always be solved by use of the general Scales 2,3, 5 and 6, but that the special isothermal Scales 7, 8 and 9 are more convenient.

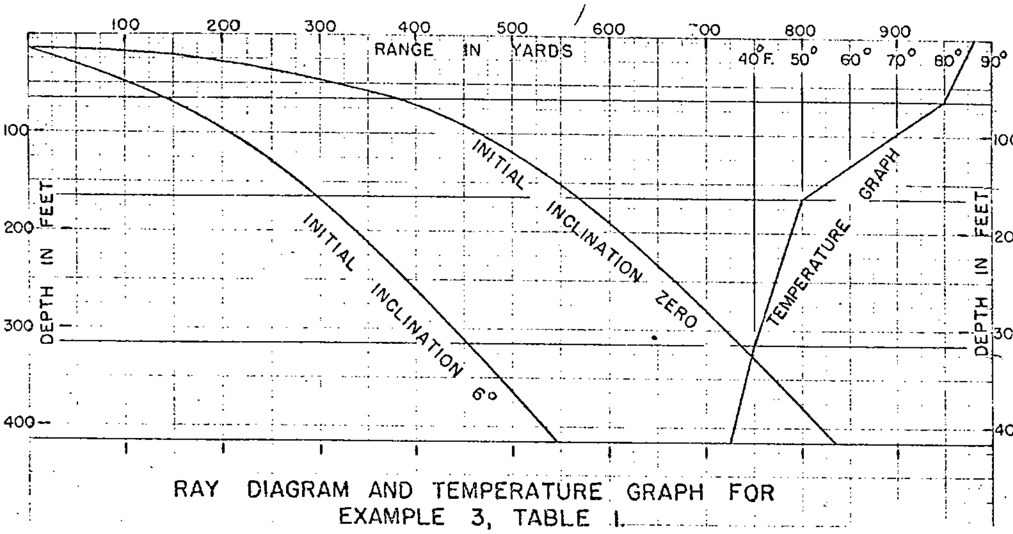
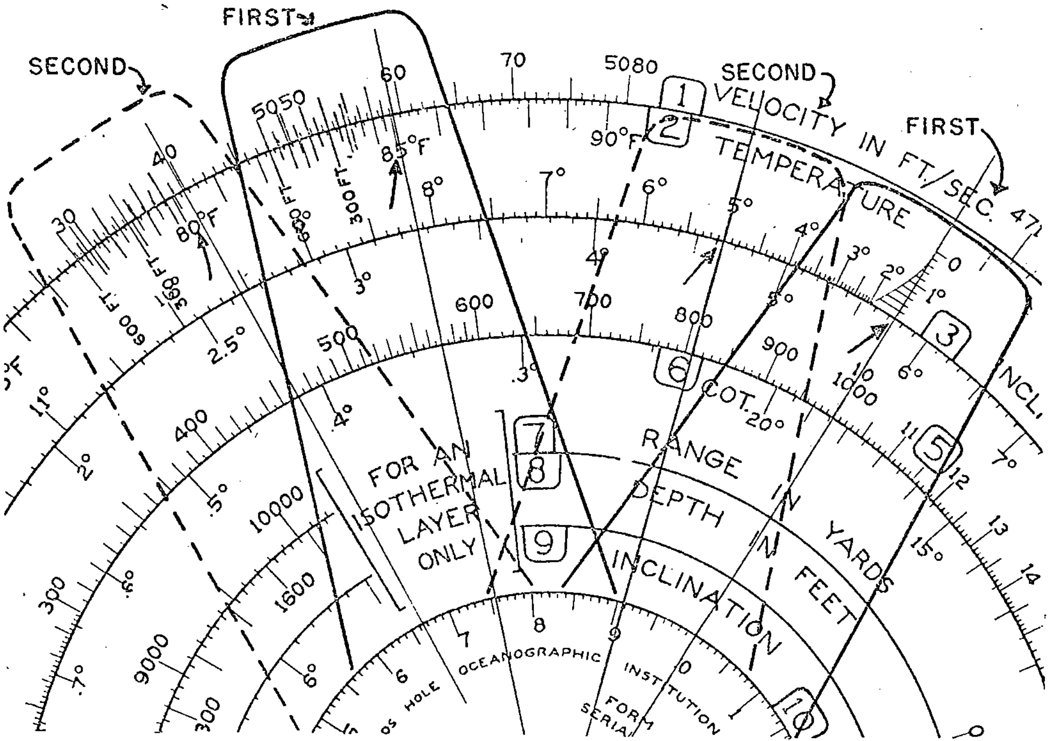


Figure 18



CALCULATION OF INCLINATION

$h_0 = 15$ FT. $T_0 = 85°$F. $\theta_0 = 0°$

$h_1 = 65$ FT. $T_1 = 80°$F. $\theta_1 = 5.02°$

Figure 19

B. Examples on Downward Refraction.

Example 3. Trace the rays leaving the projector at 0° and at 6° for the thermal distribution in the water shown by the data in columns 1 and 2 of Table I and Figure 18. Projector depth is 15 feet.

Solution: Depths and temperatures are tabulated in Columns 1 and 2. Set the long arm of the slide rule so that 15 feet depth on that arm coincides with 85° F on Scale 2, and set the short index arm on 0° on Scale 3.

TABLE I

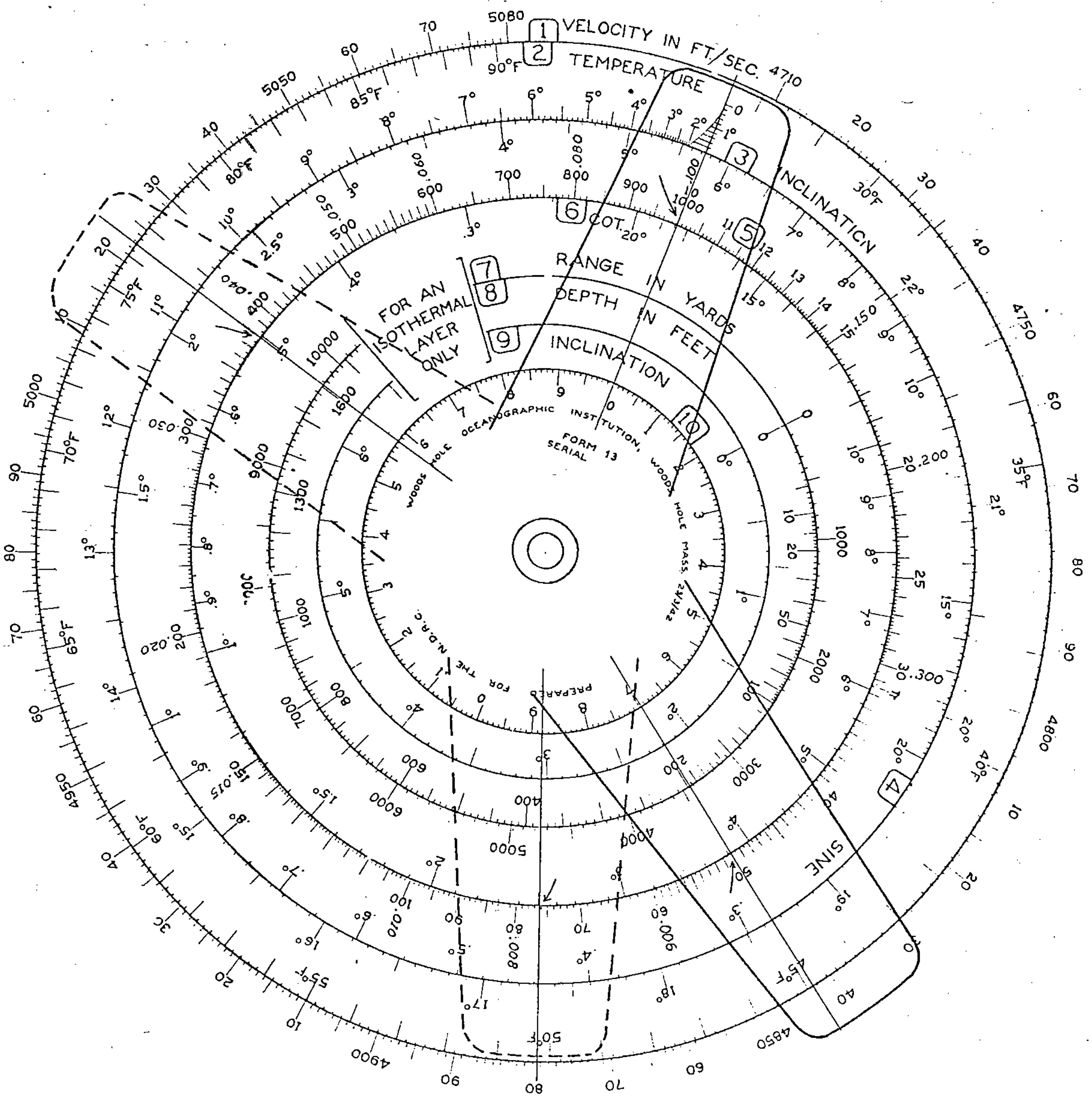
Sample Ray Diagram Calculation for Figure 18. Example 3.

(1) Depth h ft	(2) Temp. T °F	(3) Incl. θ –	(4) Mean Incl. $\bar{\theta}$ –	(5) Depth Dif. Δh ft	(6) Part. Range ΔX yd.	(7) Range X yd.
15*	85*	0*	–	–	–	–
65	80	5.02	2.51	50	380	380
165	50	15.26	10.14	100	187	567
315	40	17.99	16.62	150	168	735
415	35	19.33	18.66	100	99	834
15*	85*	6.00*	–	–	–	–
65	80	7.82	6.91	50	138	138
165	50	16.37	12.10	100	156	294
315	40	18.93	17.65	150	157	451
415	35	20.19	19.56	100	94	545

*Initial settings.

Keep the angle between the two index arms fixed for all subsequent calculations on this ray. Move the long index arm so 65 feet depth coincides with 80°F, and read the inclination 5.02° under the short index arm. This calculation is illustrated in Figure 19. Repeat this operation for each depth and temperature in Columns 1 and 2, obtaining the inclinations entered in Column 3.

Column 4 contains the mean inclination of the ray in each layer, obtained by averaging the inclinations at the top and bottom of the layer. Column 5 contains the thickness in feet of each layer, obtained by subtraction of successive values in Column 1. Column 6 contains the partial range for each layer, obtained from the corresponding entries in Columns 4 and 5 as follows: set the long index on 50 feet on Scale 5 and the



SAMPLE CALCULATION FOR TABLE I

Δh = 50 FT. θ = 2.51 ΔX = 380 YDS.

50 COT 2.51 = 380

Figure 20

short index on the initial point of the same scale (marked 10, 1000). Keeping the included angle constant, move the long index until the short index is at 2.51° on Scale 6 and read 380 yards under the long index on Scale 5. This calculation is illustrated in Figure 20. Column 7 contains the ranges of the given ray for each depth, and these values are obtained by summing the partial ranges of Column 6.

The process is repeated for the ray leaving the projector at an inclination of 6°, by setting the short index arm initially at 6° instead of 0° (See Figure 8). This example illustrates the use of the fundamental scales of the slide rule.

Example 4. For a projector depth of 15 feet and a sounding of 50 fathoms compute the limiting rays of the beam, the temperatures and depths being those given in Table II and Figure 21. Include the beam reflected from the bottom.

Solution: Since the refraction is always downward in this problem, the upper limit of the beam is the ray just tangent to the surface. The "lower limit" of the beam will depend on the beam pattern, which is different for different echo-ranging equipment. The intensity of sound emitted along any ray is usually a complicated function of the angle which that ray makes with the beam axis. For rays inclined at more than 6° to the axis, however, the emitted intensity is usually so low that it may be neglected. It is, therefore, frequently convenient to take a 6° ray as the lower limiting ray of the beam, keeping in mind that this is a relatively crude approximation, and in cases where accurate information is desired, the beam pattern of the equipment used must be taken into account.

We may, therefore, proceed to calculate the two "limiting rays" for this problem. The ray tangent to the surface is calculated first; the special scale on the reverse side of the Slide Rule may be used to obtain that part of the ray which lies above the projector level. By setting the index at 70°F on the scale for a projector depth of 15 feet and taking a temperature difference of 0.3° F between projector and surface, one obtains the value R= 456 yards and Θ = 1.26°, as shown in Figure 22. This means that the ray which leaves the projector at an angle of 1.26° above the horizontal becomes tangent to the surface at a distance of 456 yards. It is clear from the symmetry of the path that this ray is inclined 1.26° below the horizontal at twice this distance, or 912 yards.

TABLE II

Calculations for Figure 21, Example 4.

(1) h ft.	(2) T °F	(3) θ –	(4) $\bar{\theta}$ –	(5) Δh ft.	(6) $\Delta\bar{X}$ yd.	(7) X yd.
15**	69.7**	-1.26**	REVERSE	RULE	-	0
0	70.0**	0.00	"	"	456**	456
15*	69.7**	1.26*	"	"	456	912
100	68.0	3.33	2.30	85	708	1620
200	40.0	15.55	9.44	100	202	1822
300	38.0	16.13	15.84	100	118	1940
200	40.0	15.55	15.84	100	118	2058
100	68.0	3.33	9.44	100	202	2260
15	69.7	1.26	2.30	85	708	2968
0	70.0	0.00	REVERSE	RULE	456	3424
15*	69.7*	6.00	-	-	-	0
100	68.0	6.72	6.36	85	255	255
200	40.0	16.60	11.66	100	162	417
300	38.0	17.13	16.86	100	110	527
200	40.0	16.60	16.86	100	110	637
100	68.0	6.72	11.66	100	162	799
15	69.7	6.00	6.36	85	255	1054

*Initial settings, Scales 2 and 3.
**Initial settings, reverse side of rule.

The remaining calculations are exactly similar to those of Example 3, the proper initial settings on Scales 2 and 3 for this step being 69.7°F at 15 feet depth, with the inclination 1.26°.

The algebraic signs of the inclinations θ in Table II illustrate the rule of signs which must be applied in all cases where intensity calculations are to be made. Prior to reflection from either surface or bottom, O is positive at any point where the ray is descending and negative where it is ascending. After each reflection this rule is reversed. Thus, after 1, 3, or 5 reflections θ is negative on descending and positive on ascending rays. After 0, 2, or 6 reflections, θ is positive on descending and negative on ascending rays. Thus, for any particular ray, passage through a vertex changes the sign of the inclination, but reflection from surface or bottom does not.

The calculations for the 6° ray are also given in Table II and need no further comment. The ray diagram in Figure 21 shows the two rays calculated in this problem as heavy lines, each with its initial angle written on it, and includes other rays sketched in to indicate the extent of the beam.

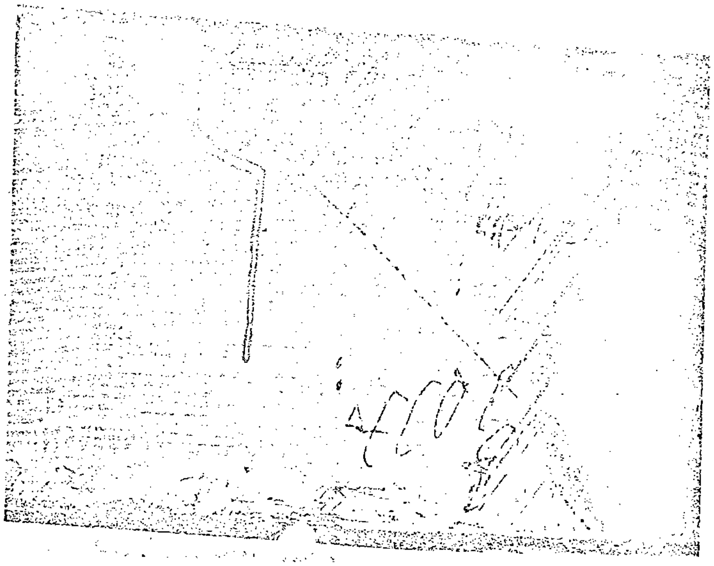
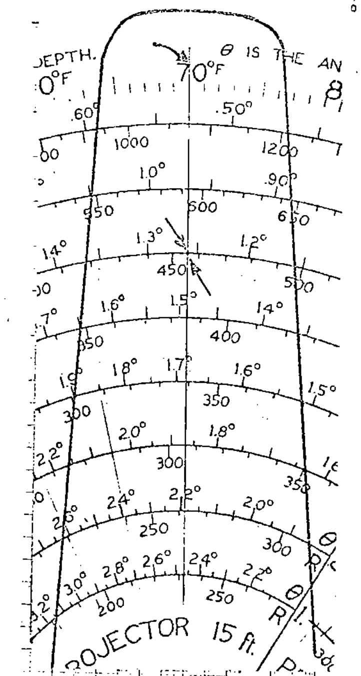


Figure 23



INITIAL SLIDE RULE SETTING FOR TABLE II

T_0 = 70°F T_1 = 69.7°F

PROJECTOR DEPTH = 15 FT.

INITIAL INCLINATION = 1.26°

RANGE TO SURFACE = 456 YDS.

Figure 22

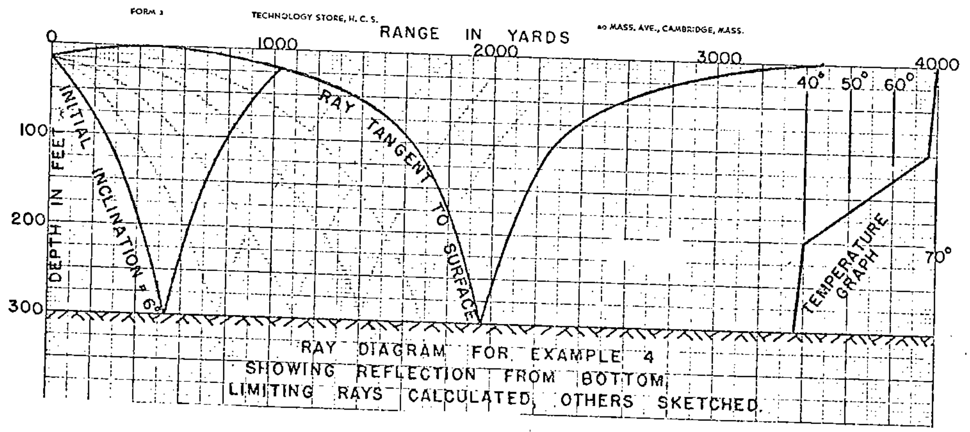


Figure 21

C. Examples On Upward Refraction:

Example 5. Interpret bathythermograph slide #4101-f; 16-II-41; 39-24 N; 69-05 W, which is shown in Figure 23. The depth scale on this slide is in fathoms. Projector depth is 15 feet.

Solution: As shown in Figure 23, the water is isothermal at 50.6°F to a depth of 14.5 fathoms. The temperature then increases uniformly to 56.0°F at 21 fathoms and then remains isothermal to 58 fathoms, the maximum depth reached by the instrument. No effort was made to read the temperature from the BT slide with extreme accuracy. The readings were simplified to give an example illustrating a certain point. This temperature distribution is characteristic of the winter condition in which fresher shelf water overlies the warmer but more saline slope water just beyond the edge of the continental shelf. The salinity of the warmer water is known to be great enough to make it more dense than the water above it.

The chief effect of the salinity distribution is, therefore, to stabilize the observed temperature distribution, which otherwise could not exist. The salinity distribution also has an effect on the refraction pattern; this has not been considered since data are usually not available, and the effect is usually not very large. The relatively high salinity of the lower layer increases the velocity difference between the two layers and makes upward refraction somewhat sharper than is calculated from the temperature distribution alone. If the salinity is known, the exact refraction pattern may be computed by the method given on Page 26 .

Since the temperature in this problem never decreases with depth, it is clear at once that the refraction is always upward. The three most significant rays to trace are, therefore, those tangent to the bottom of each of the three layers. These are the first three rays calculated in Table III, and the ray diagram for them is Figure 24. The first ray is calculated on the isothermal scales alone, by setting on 72 feet and then on 87 feet on Scale 8, and proceeding as in Example 2, Page 16 .

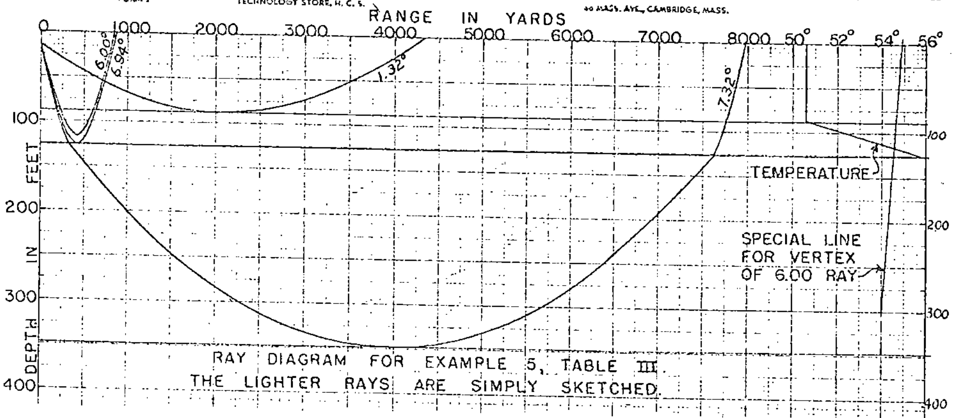


Figure 24

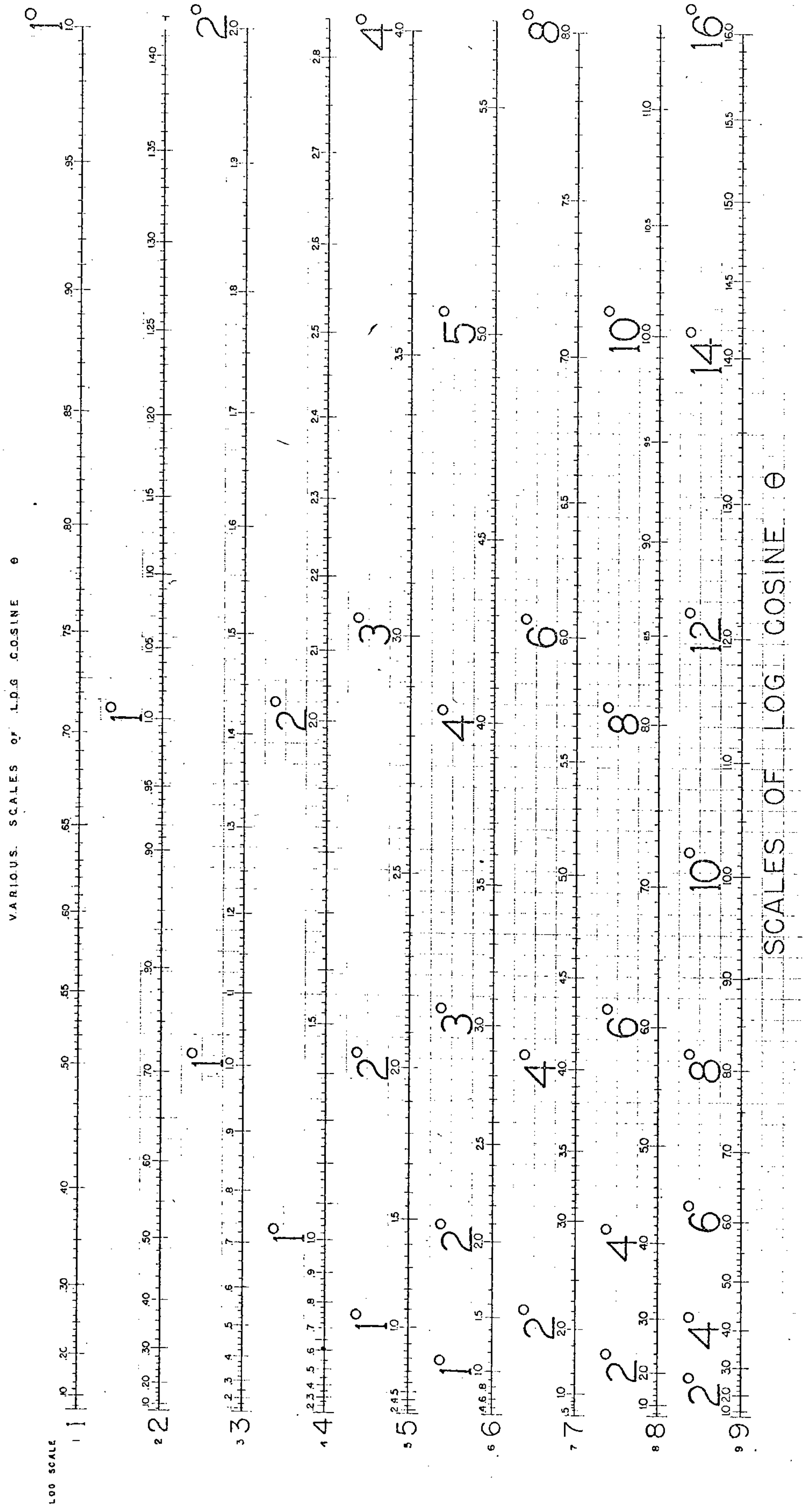

VARIOUS SCALES OF LOG COSINE θ
LOG SCALE
SCALES OF LOG COSINE θ
1 2 3 4 5 6 7 8 9
1° 2° 4° 8° 16°
1° 2° 3° 4° 5° 6° 8° 10° 12° 14°

Figure 25

TABLE III

Calculations for Figure 24, Example 5.

(1) h ft.	(2) T °F.	(3) θ -	(4) $\bar{\theta}$ -	(5) Δh ft.	(6) ΔX yd.	(7) X yd.
15	50.6	+1.32**	-	-	-	0
87	50.6	0	ISOTHERMAL	72**	2080**	2080
0	50.6	-1.44	"	87	2280	4360
15	50.6	+6.94	-	-	-	0
87	50.6	+6.82	+6.88	72	200	200
126*	56.0*	0 *	+3.41	39	219	419
87	50.6	-6.82	-3.41	39	219	638
15	50.6	-6.94	-6.88	72	200	838
0	50.6	-6.96	-6.95	15	41	879
15	50.6	+7.32	-	-	-	0
87	50.6	+7.22*	+7.27	72	188	188
126*	56.0*	+2.32**	+4.77	39	156	344
348	56.0	0	ISOTHERMAL	222**	3630**	3974
126	56.0	-2.32	"	222	3630	7604
87	50.6	-7.22	-4.77	39	156	7760
15	50.6	-7.32	-7.27	72	188	7948
0	50.6	-7.36	-7.34	15	39	7987
15*	50.6*	+6.00*	-	-	-	0
87	50.6	+5.88	+5.94	72	230	230
116	-	0	+2.94	29	189	419
87	50.6	-5.88	-2.94	29	189	608
15	50.6	-6.00	-5.94	72	230	838
0	50.6	-6.02	-6.01	15	47	885

---

*Initial settings, Scales 2 and 3
**Initial settings, Scales 7, 8 and 9

The rays which were actually calculated are shown in Figure 24 by heavy lines with the initial angle of each written on it. The rays indicated by light lines are merely sketched in to aid in visualization of the structure of the sonic beam. As in preceeding examples, we will calculate the ray leaving the projector at an angle of 6° below the horizontal, which may be thought of as the lower limit of the beam in case the boat is not rolling or pitching.

The calculation of the 6° ray introduces one new situation. To the bottom of the isothermal layer the ray may be traced either by an initial setting of 15 feet, 50.6°F, and 6.00° on Scales 2 and 3, or by using the special isothermal Scales 7, 8 and 9 and following the method of Example 2. Either method shows that the range to the bottom of the layer is 230 yards and that the inclination at this point is about 5.88°. If the attempt is made to find the inclinations of this ray at the bottom of the second layer, the index for inclination goes off scale, indicating that the ray does not penetrate to this depth. As the whole system of the slide rule is based on the assumption that the temperature and depth are known for both ends of each segment of the ray, it is necessary to determine in advance the depth which this penetrates into the second layer before becoming horizontal. To do this, retain the original angle between the arms of the slide rule obtained from the setting 50.6°F, 15 feet and 6.00°, and move these so that inclination indicated is zero. Now the depth scale on the long arm, when used with Scale 2 in this setting, shows all possible combinations of temperature and depth which would give the proper velocity to render this ray horizontal. To determine the one of these combinations which actually exists in this case, it is best to construct a graph of the indications of the long arm, superimposed on the given temperature-depth graph of Figure 24. From the long arm of the slide rule, we read 54.9°F at a depth of zero, and 54.0°F at a depth of 300 feet. The straight line connecting these two points in Figure 24 cuts the temperature depth graph at the depth of 116 feet, which is therefore the depth at which the 6.00° ray becomes horizontal. Putting this depth and inclination in Table III permits completion of the calculation of the 6.00°ray.

The extra step which had to be taken in the above calculation may sometimes be avoided by taking as initial data the depth to which the ray is to go instead of the initial angle of the ray.

By inspection of Figure 24 it is seen that as the initial angle changes from 6.00° above the horizontal to 1.32° below, the range (on a surface target) increases from a very small value up to 4360 yards. At this point the ray just reaches the bottom of the isothermal layer, and as the angle below the horizontal increases further, the range decreases sharply, due to the strong refraction in the second layer. The minimum range occurs for an angle in the neighborhood of 6.00° and is only about 885 yards. If the axis of the beam at the projector were strictly horizontal, and if no sound left the projector at angles greater than 6° with the axis it would not be necessary to consider any further rays, but in cases such as the present one, where a slight increase in the angle can cause a great change in the range, it is advisable to trace a few additional rays in order to see what a roll of even a few degrees would do to the sonic beam. After the rays begin to enter the third layer, first at an angle of 6.94°, the range increases rapidly due to the very slight refraction in this isothermal layer. At an initial angle of 7.32° the range to a surface target is up to 7987 yards.

It should be noted that the sound transmitted through the third layer may be expected to be much weaker than sound transmitted an equal distance through a sufficiently thick isothermal layer at the surface. It may be seen from the diagram that the energy which leaves the projector between the angles 6.94° and 7.32° is spread over the surface between the ranges 879 and 7987 yards.

D. Examples on Use of the Velocity Scale.

Example 6. Illustrate the use of Scale 1 of the slide rule.

Solution: (a) To determine the velocity of sound in water at a given temperature and depth, set the long arm of the rule so the given depth coincides with the given temperature as read on Scale 2. Read the velocity under the index of the long arm on Scale 1. (For illustration see Figure 7.)

(b) To trace a ray when the given data consists of velocity and depth instead of temperature and depth, proceed in the standard way for tracing rays except that the main index of the long arm is set on the given velocity on Scale 1; inclinations are then read under the index of the short arm on Scale 3 in the usual way. In this case, the depth on the long arm is not used, since the depth correction has already been put in to obtain the given velocities. This method of calculation is useful when changes in salinity need to be taken into account. An increase in salinity of 1 part per thousand increases the velocity of sound 4.3 feet per second.

(c) An alternative method of taking account of changes in salinity, without use of the velocity scale, is to note that an increase of one part per thousand in salinity increases the velocity as much as an increase in depth of 236 feet. Thus, if all salinities below a certain level exceed those above that level by 0.2 parts per thousand, we may increase all depths in the more saline water by 47 feet obtaining "equivalent depths" which are set on the depth scale of the long index arm and Scale 2 in the usual way. The equivalent depths are used only in the calculation of inclination, and the true depths are used to obtain h. As pointed out in the basic assumptions, salinity seldom has to be considered in practice, but may be taken into account as above when known.

E. Method of Proportions.

1. Purpose of the Method.

The slide rule, when used as described above, provides all the precision justified in view of the limitations in the thermal data and the deviations from strictly horizontal stratification of sea water. However, there are cases in intensity calculation where it is desirable to determine inclinations and ranges with greater accuracy in order to obtain a self-consistent result. This situation usually arises with a series of rays which have vertices at varying depths in a layer having a fairly large thermal gradient. In this case it is recommended that the ray whose vertex just reaches the far side of the layer be first calculated by the standard method, and that all other rays which penetrate this layer less deeply be calculated by the method of proportions.

2. Application to a Layer Containing a Vertex.

For inclinations less than about 10° it is permissible to assume that: the range X is proportional to the inclination with which the ray enters the layer and the depth of penetration is proportional to the square of this angle, and thus to the square of X. For example, in Table III and Figure 24, consider how the range and penetration for the bottom segment of the 6.00° ray could have been obtained from those of the 6.94° ray. The ratio of entering inclinations is (at 87 feet depth) 5.88/6.82=.860, (Figure 11). Thus the range is .860 x 219=188 yards,(Figure 11), and the penetration is $(.860)^2 \times 39 = 29$ feet, values which are in excellent agreement with those obtained by the earlier method. It will be noted that the method of proportions is considerably faster, and, for small angles, much more accurate than the other method.

3. Application to Other Layers.

Suppose one ray to have been calculated by the standard method and found to have inclinations Θ_0 and Θ_n at the projector, the inclination $\Theta_n + \Delta\Theta_n$ at the second level is given by Snell's Law, Equation 1, as

$$\text{logcos}\,(\Theta_n+\Delta\Theta_n) - \text{logcos}(\Theta_0+\Delta\Theta_0) = \text{logcos}\Theta_n - \text{logcos}\,\Theta_0 \quad (3)$$

In Figure 25 there are a number of log cosine scales which may be used in this calculation. For example, derive the 6.00° ray in Table III (Page24)from the 7.32° ray. Place the edge of a piece of paper on Scale 7 of Figure 24, and mark at 7.32° and at 6.00°. It may be seen in Table III that the inclination of the 7.32° ray becomes 7.22° at a depth of 87 feet. If the paper is shifted so that the mark made at 7.32° reads 7.22°, it will be found that the mark at 6.00° now reads 5.88°, which is therefore the inclination of the 6.00° ray at a depth of

87 feet. Repetition of this check for other pairs of values should give reasonably good agreement in all cases. Obviously, a pair of dividers could be used for this purpose, and another scale could be selected for angles not lying within the range of Scale 7.

Scale 3 of the slide rule is a log cosine scale, and may also be used for this type of calculation. For angles too small to be accurately read on this scale, we may consider the graduations to be multiplied by any convenient factor to provide the desired range of values. Thus the scale may be considered to run from 0° to 2.2°, 4.4°, 6.6°, 11°, etc.

Suppose one ray, which will be designated by the letter A, has been calculated by the standard method, and a second ray, B, is to be determined. Set the long arm to read θ_{oA}, the initial inclination of ray A, and the short arm to read θ_{oB}, that of ray B, both on Scale 3 of the slide rule. Maintain the angle between the arms, set the long index on θ_{1A}, the inclination of ray A at the bottom of the first layer, and read θ_{1B} under the short index. Repeat for all layers,

Example 7. A common situation is that in which isothermal water overlies a negative temperature gradient. In a typical case the water temperature was 80°F from the surface to 100 ft. and below this decreased at a rate of 4°F per 100 feet for at least 300 feet. The projector was assumed to be at the surface.

Solution: The limiting ray is obtained by setting at 100 feet on Scale 8 and reading 2430 yards and 1.56° on Scales 7 and 9. As this ray started from a projector at the surface it will reach the surface again, by symmetry, at 4860 yards. From Scales 2 and 3 this ray whose inclination is zero at 80°F and 100 feet depth is found to have an inclination of 8.12° at 68°F and 400 feet and inclinations at other depths as shown in Table IV. The inclination at each depth shown in Table IV of the ray leaving the projector at 6.00° is found by setting at 80°, 0 feet, and 6.00°. The inclinations of all other rays at all depths are obtained from those for the 6.00° ray by means of Figure 25.

In this case, the calculation of each ray directly from temperature and depth data would be much more laborious. This detailed calculation is unnecessary for a ray diagram, but is required as a basis for intensity calculations as shown in the next section.

TABLE IV

Calculations for Example 7 and Figure 28.

(1) h ft.	(2) T °F.	(3) Θ -	(4) $\bar{\Theta}$ -	(5) Δh ft.	(6) ΔX yd.	(7) X yd.
0	80	1.56	-	-	-	0
100	80	0.00	ISOTHERMAL		2430	2430
110	79.6	1.46	0.73	10	262	2692
125	79	2.30	1.88	15	152	2844
150	78	3.26	2.78	25	172	3016
200	76	4.60	3.93	50	244	3260
250	74	5.64	5.12	50	187	3447
300	72	6.51	6.08	50	157	3604
350	70	7.52	6.89	50	137	3741
400	68	8.12	7.62	50	125	3866
0	80	1.57	-	-	-	0
100	80	0.18	ISOTHERMAL		2180	2180
110	79.6	1.47	0.82	10	233	2413
125	79	2.31	1.89	15	152	2565
150	78	3.27	2.79	25	172	2737
200	76	4.60	3.94	50	244	2981
250	74	5.64	5.12	50	187	3168
300	72	6.51	6.08	50	157	3325
350	70	7.27	6.89	50	137	3462
400	68	7.96	7.62	50	125	3587
0	80	1.60	-	-	-	0
50	80	1.17	1.38	50	695	695
100	80	0.42	.80	50	1200	1895
110	79.6	1.65	1.04	10	184	2079
125	79	2.24	1.94	15	148	2227
150	78	3.20	2.72	25	176	2403
200	76	4.60	3.90	50	246	2649
250	74	5.64	5.12	50	187	2836
300	72	6.54	6.09	50	157	2993
350	70	7.31	6.92	50	138	3131
400	68	8.13	7.72	50	123	3254
0	80	1.75	-	-	-	0
50	80	1.37	1156	50	618	618
100	80	0.82	1.10	50	871	1489
110	79.6	1.79	1.30	10	147	1636
125	79	2.34	2.06	15	140	1776
150	78	3.28	2.81	25	170	1946
200	76	4.65	3.96	50	242	2188
250	74	5.69	5.17	50	185	2373
300	72	6.58	6.14	50	155	2528
350	70	7.34	6.96	50	137	2665
400	68	8.16	7.75	50	123	2788

TABLE IV (continued)

(1) h ft.	(2) T °F.	(3) Θ –	(4) $\bar{\Theta}$ –	(5) Δh ft.	(6) ΔX yd.	(7) X yd.
0	80	2.00	–	–	–	0
50	80	1.70	1.85	50	512	512
100	80	1.30	1.50	50	630	1142
110	79.6	2.03	1.66	10	116	1258
125	79	2.53	2.28	15	126	1384
150	78	3.38	2.96	25	162	1546
200	76	4.70	4.04	50	234	1780
250	74	5.76	5.23	50	181	1961
300	72	6.64	6.20	50	153	2114
350	70	7.42	7.03	50	134	2248
400	68	8.22	7.82	50	121	2369
0	80	2.50	–	–	–	0
50	80	2.26	2.38	50	400	400
100	80	2.00	2.13	50	445	845
110	79.6	2.54	2.27	10	84	929
125	79	2.96	2.75	15	108	1037
150	78	3.72	3.34	25	143	1180
200	76	4.96	4.34	50	220	1400
250	74	5.96	5.46	50	174	1574
300	72	6.82	6.39	50	148	1722
350	70	7.58	7.20	50	134	1856
400	68	8.34	7.96	50	129	1985
0	80	4.00	–	–	–	– 0
50	80	3.82	3.91	50	244	244
100	80	3.70	3.76	50	254	498
110	79.6	4.03	3.86	10	49	547
125	79	4.30	4.16	15	70	617
150	78	4.80	4.55	25	105	722
200	76	5.86	5.33	50	180	902
250	74	6.70	6.29	50	152	1054
300	72	7.48	7.09	50	134	1188
350	70	8.18	7.83	50	121	1309
400	68	8.90	8.54	50	111	1420
0	80	6.00	–	–	–	– 0
50	80	5.92	5.96	50	161	161
100	80	5.80	5.90	50	162	323
110	79.6	6.01	6.00	10	32.2	355
125	79	6.20	6.00	15	47	402
150	78	6.60	6.40	25	75	477
200	76	7.34	6.97	50	137	614
250	74	8.06	7.70	50	124	738
300	72	8.70	8.38	50	113	851
350	70	9.32	9.01	50	105	956
400	68	9.94	9.63	50	98	1054

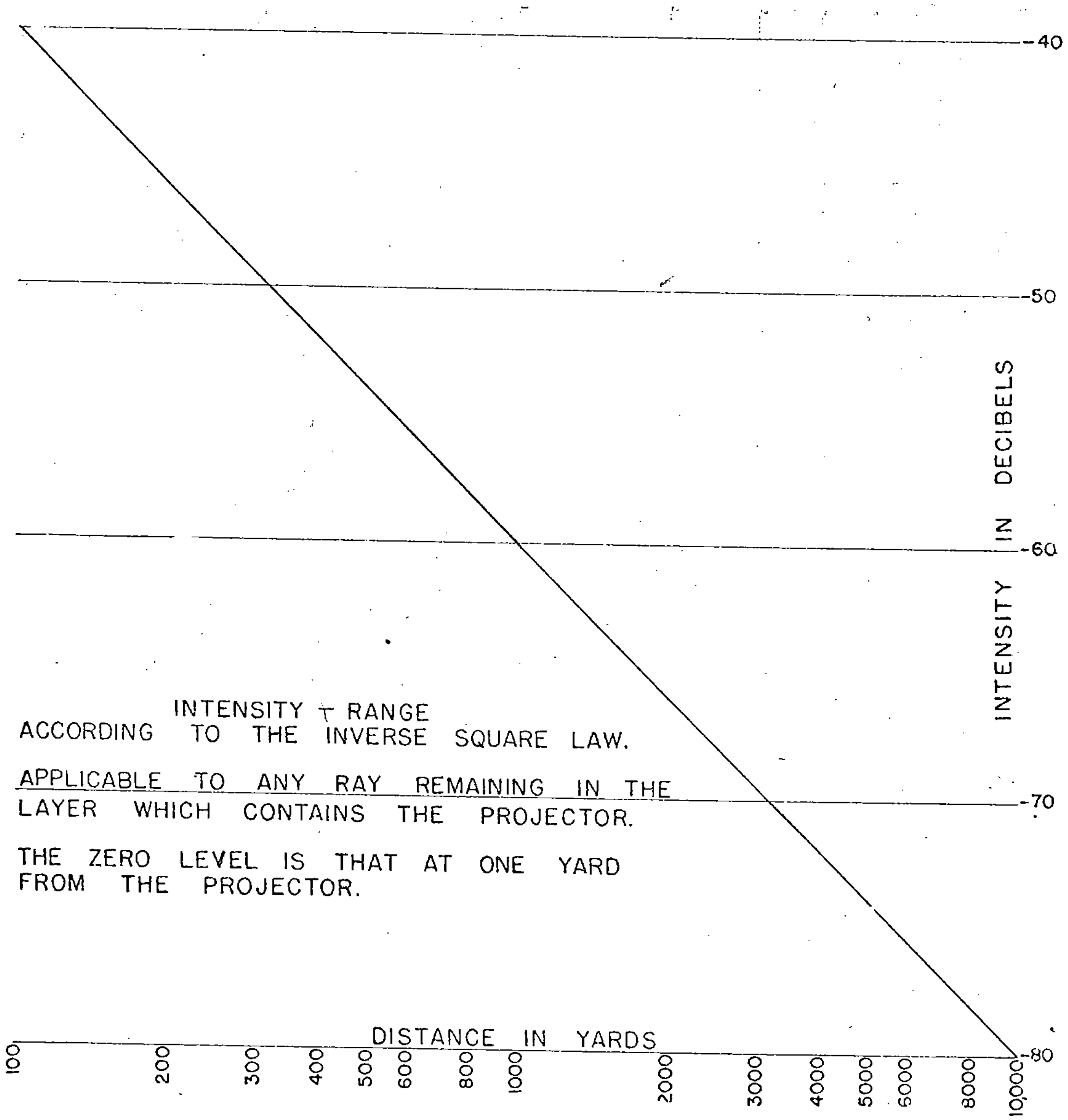

-40
-50
-60
-70
-80
INTENSITY IN DECIBELS
INTENSITY ┬ RANGE
ACCORDING TO THE INVERSE SQUARE LAW.
APPLICABLE TO ANY RAY REMAINING IN THE LAYER WHICH CONTAINS THE PROJECTOR.
THE ZERO LEVEL IS THAT AT ONE YARD FROM THE PROJECTOR.
DISTANCE IN YARDS
100
200
300
400
500
600
800
1000
2000
3000
4000
5000
6000
8000
10,000

Figure 26

III INTENSITY CALCULATIONS

A Introduction

Not all the characteristics of the sound beam become apparent from a glance at the ray diagram. Although the general distribution of intensity is governed by the spreading of the rays in the different parts of the beam, the extent of spreading is not easy to judge. Methods exist for the direct calculation of intensity in different parts of the beam and advantages will arise from their use when possible.

1. When the distribution of intensity is known in detail, it is possible, by observing the maximum range at which a target of known position can be detected, to determine the efficiency of an individual installation. If, for instance, under the conditions shown in Figure 29B, a surface target can just be detected at 1800 yards, the installation will be unable to detect a target at any point where the intensity is less than -65 decibels.

2. By the accumulation of such data it is possible to allow for the effects of the searching vessel's speed, and wind on the maximum range at any depth.

3. Such effects as "echoing off the Gulf Stream", when the intensity in some part of the beam is so high that even a feeble reflection will give a loud echo, will be predictable.

The methods of calculating intensities which are applicable to every beam pattern are laborious. A method is given below which is useful in most of the common cases, and a number of rules and typical diagrams are given which will make clear the approximate situation in nearly all cases.

B. Graphical Determination

In accordance with the principles discussed on page 6 it is possible to determine graphically the average intensity between two rays of the sound field. Since in most cases the angle which a ray makes with the horizontal is quite small, a plane perpendicular to any ray will be very nearly vertical. Thus the vertical distance between two adjacent rays is a measure of the area(in the cross section) over which the energy between those two rays is spread. This vertical distance, which may be called H, can therefore be used to calculate the average intensity between the two rays.

To apply this method, let A represent the initial angle between two rays. If there were no refraction and the rays were straight lines, at a horizontal range X from the projector the distance between these two rays would be X sin A. When refraction is present, the actual average intensity at this range equals the inverse square value multiplied by (X sin A)/H.

This method can be applied only if all intermediate rays fall between the two rays used for the calculation. Whenever the velocity depth curve is concave to the right, the rays from some sector of the beam cross rays from some other sector, leaving areas in which rays from widely separated sectors of the beam are thinly scattered. In Figure 24, for instance, it would be i possible to calculate the intensity at 4000 yards using the 1.32° ray and the 7.32° ray because very few of the rays between 1.32° and 6.94° fall in this area. If the 7.25° ray were plotted, however, it would be possible to calculate the intensity at 4000 yards between this ray and the 7.32° ray because all intermediate rays lie between these two.

This method has the disadvantage that it requires plotting a substantial number of rays, usually at least ten or fifteen, and in some cases as may as twenty or thirty. For this purpose it is simplest to compute three or four rays accurately from Equation 1 and then to fill in the other rays by the Method of Proportions described in section II-E on page 27. When an accurate ray diagram has been obtained in this way; A and H can be determined at any range X and at any depth. When these quantities are known, the intensity ratio in decibels may be found directly from Figure 27.

In two types of ray diagrams the application of the Graphical Method is especially direct. For a bundle of rays which never leave the layer of constant velocity gradient which contains the projector, the inverse square law holds and the intensity at any horizontal range from the projector can be found directly from Figure 26.

When the velocity of sound at the depth of the projector is less than that above and below it, a sound channel exists. There is in such a case some velocity which is the maximum velocity occurring both above and below the projector and the sound channel lies between the two depths at which this velocity is found. The rays tangent to the two levels leave the projector at the same angle but with opposite signs and these and all rays between them travel between the two levels indefinitely. Other rays may lie within the sound channel for a short distance but are soon refracted out and lost. On the average the rays between the limiting rays are uniformly distributed between the top and bottom of the sound channel.

If the two rays bounding the sound channel have the initial inclinations +A/2 and -A/2, the initial angle between them is A. Letting H represent the difference in depth of the upper and lower limits of the channel, A/H can be calculated, and the intensity of sound at any range found from Figure 26.

<u>Example 8</u> Intensities in the ray diagram calculated in Example 7, Page 28, Figure 28. The data are shown in Table V.

The intensities in the isothermal layer at several convenient ranges are obtained from figure 26, and the intensity contours are vertical lines down to the bottom of the isothermal layer. The intensity distribution in the layer of constant velocity gradient containing the projector would be found and plotted in the same way, whatever the gradient.

To construct the table for the calculation of intensities below the isothermal layers set down in columns (1) and (2) the initial inclinations Θ_1 and Θ_2 of the first two rays. In column (6) set down the difference A, between these quantities. In column (9) set down convenient values determined by inspection of the ray diagram of the range, X, at which the intensities between the rays Θ_1 and Θ_2 will be calculated. From the ray diagram read h_1 and h_2, the depths to the rays Θ_1 and Θ_2 at the range X. In column (5), $H=h_1-h_2$ and in column (10), the mean depth of the rays Θ_1 and Θ_2 at the range X, $h=(h_1+h_2)/2=h_2+H/2$. The values of A/H in column (7) are obtained by using scale (5) of the slide rule according to the method of division described on page 11. The values of the intensity of sound I are found in figure 2 by following the slanting line for A/H until it intersects the vertical line representing the corresponding range, X, and reading the intensity of sound in decibels on the horizontal line passing through this intersection.

Plot the points $(X,\overline{h})$ on the ray diagram and beside each one write the corresponding intensity in even decibels. The intensity contours may then be sketched in.

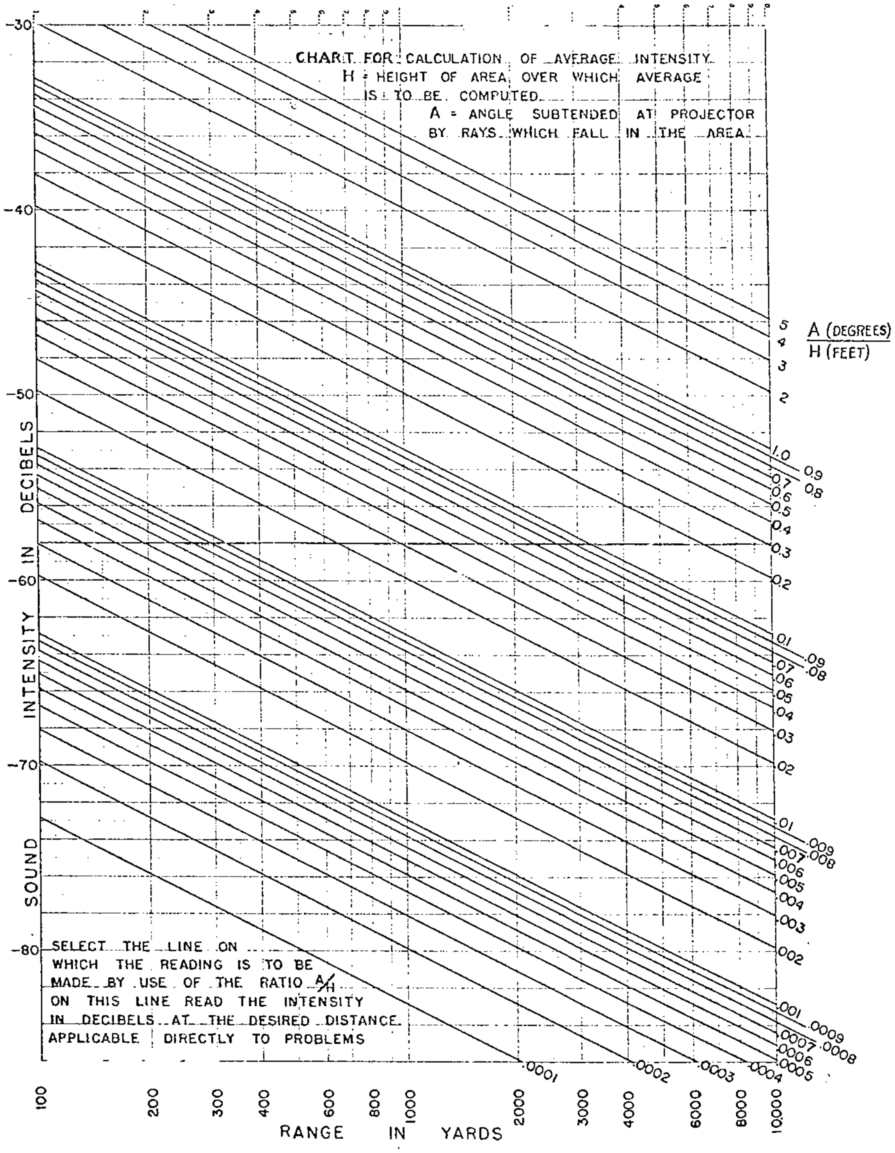


CHART FOR CALCULATION OF AVERAGE INTENSITY
H = HEIGHT OF AREA OVER WHICH AVERAGE IS TO BE COMPUTED
A = ANGLE SUBTENDED AT PROJECTOR BY RAYS WHICH FALL IN THE AREA
SELECT THE LINE ON WHICH THE READING IS TO BE MADE BY USE OF THE RATIO A/H
ON THIS LINE READ THE INTENSITY IN DECIBELS AT THE DESIRED DISTANCE
APPLICABLE DIRECTLY TO PROBLEMS
SOUND INTENSITY IN DECIBELS
-30
-40
-50
-60
-70
-80
RANGE IN YARDS
100
200
300
400
600
800
1000
2000
3000
4000
6000
8000
10000
A (DEGREES) / H (FEET)
5
4
3
2
1.0
0.9
0.8
.07
.06
.05
0.4
0.3
0.2
.01
.09
.08
.07
.06
.05
.04
.03
.02
.01
.009
.008
.007
.006
.005
.004
.003
.002
.001
.0009
.0008
.0007
.0006
.0005
.0004
.0003
.0002
.0001

Figure 27

TABLE V

Intensity Calculation for Figure 26

(1) initial inclinations	(2)	(3) depths to rays	(4)	(5) vertical distance between rays	(6) angle between rays	(7)	(8) sound intensity	(9) range	(10) mean depth
θ_1	θ_2	h_1	h_2	H	A	A/H	I	X	$\bar{h}$
degrees	degrees	Feet	Feet	Feet	degrees	-	decibels	yards	Feet
6.00	4.00	78	51	27	2.00	.074	48.6	250	65
		157	100	57		.035	54.6	500	129
		254	157	97		.021	58.2	750	209
		320	201	119		.017	60.2	900	260
4.00	2.50	100	61	39	1.50	.039	54.0	500	81
		157	89	68		.022	58.0	750	128
		230	120	110		.014	61.6	1000	175
		323	167	156		.0097	64.2	1250	245
2.50	2.00	61	49	12	0.50	.042	53.6	500	55
		89	70	19		.026	57.6	750	80
		120	89	31		.016	61.3	1000	105
		167	110	57		.0088	64.7	1250	139
		228	143	85		.0059	66.2	1500	186
		308	193	117		.0043	67.4	1750	252
		350	218	132		.0037	68.4	1850	284
2.00	1.75	89	73	16	0.25	.016	61.3	1000	81
		110	87	23		.011	63.5	1250	99
		143	101	42		.0060	66.1	1500	122
		193	124	69		.0036	68.4	1750	159
		260	159	101		.0025	71.9	2000	210
		350	214	136		.0018	73.9	2250	282
1.75	1.60	101	88	13	0.15	.011	63.6	1500	95
		124	96	28		.0054	66.5	1750	110
		159	105	54		.0028	71.4	2000	132
		214	128	86		.0017	74.1	2250	171
		289	168	121		.0012	76.4	2500	229
		341	200	141		.0011	76.8	2650	271
1.60	1.57	128	102	26	0.03	.0012	75.7	2250	115
		168	118	50		.00060	79.1	2500	143
		224	153	71		.00043	81.9	2750	189
		300	204	96		.00031	82.7	3000	252
		339	233	106		.00028	83.5	3100	286
1.57	1.56	118	101	17	0.01	.00059	79.2	2500	110
		153	115	38		.00026	83.3	2750	134
		204	148	56		.00018	85.0	3000	176
		277	198	79		.00013	87	3250	238
		326	237	89		.00011	88	3400	282

IV. PREDICTION OF MAXIMUM ECHO RANGE.

A. Types of Procedure.

When a bathythermograph slide is used in estimation of the maximum range of a given sound gear under a given set of conditions, it is often necessary to make a quick estimate based on nothing more than a glance at the slide. In cases where more time is available it is possible to make a more thorough analysis leading to a more accurate estimation of the range. Depending upon the time available and the complexity of the water conditions, there are the following degrees of refinement in the methods available for treating the problem:

1. Glance at slide.
2. Determine "Assured Range".
3. Draw limiting rays.
4. Construct ray diagram.
5. Construct intensity diagram.
6. Allow for reflection coefficient of bottom and of the target, state of the sea, speed of own vessel, efficiency of sound gear, variations in salinity, etc.

As would be expected, the reliability of a result obtained from one of the rapid methods will depend greatly on familiarity with the more advanced methods. Consequently it is sometimes profitable to make a very detailed and complete analysis of all factors of a given situation, even though the analysis must be made after the situation has ceased to exist.

B. Glance at Bathythermograph Slide.

Even the most elementary understanding of the principle of refraction will enable a person to classify sound-ranging conditions as good or bad, in the majority of cases, with no calculation whatever. If there is a thick isothermal layer, conditions are good. If there is a strong negative gradient at projector level (and no bottom reflection) conditions are bad.

C. Determine "Assured Range".

The pamphlet "Prediction of Echo Ranges from Bathythermograph Observations" gives rapid methods applicable to all of the simpler types of thermal structure.

D. Draw Limiting Rays.

In most cases the limiting rays, that is, the rays bounding the outer shadow zone, can be calculated and drawn in a few minutes. When they are drawn, not only is the Assured Range shown with great accuracy, but also the maximum range at all depths. In Figure 29 are shown ray diagrams corresponding to several types of temperature distribution together with data which will be discussed below on the resulting intensity distribution. Until the operator has had considerable experience a few simple rules about limiting rays will be found useful.

1. When the highest temperature in the water column is at the surface, there will be either one or two limiting rays. If the BT trace is convex to the right, the ray horizontal at the surface will be the limiting ray as shown in figure 29A. If the BT trace is concave to the right there will be two limiting rays. One will be horizontal at the surface and the other will be horizontal either at the depth of the knee in the BT trace, if this is above the projector, as in Figure 29D, or at the depth of the projector, if the knee is below this depth.

2. When the highest temperature in the water column is below the surface, there will be one limiting ray which splits at the level of maximum temperature, or at the bottom of the layer of maximum temperature, as in Figures 28, 29B and 29C. As a comparison of Figure 28 and 29B shows, the deeper the maximum temperature occurs, the longer is the range at the surface; and the steeper the thermocline, the shorter are all ranges below the surface layer. When the portion of the BT trace including the point of maximum temperature is curvilinear through a depth of at least thirty feet, there will be no limiting ray but the echo intensity at the level of the maximum temperature will be small.

3. When there is a layer of maximum <u>velocity</u>, that is a layer extending downward from the depth of maximum temperature having a negative temperature gradient of about three tenths of a degree in a hundred feet, there will be one limiting ray or none. If the projector is in the isovelocity layer, the beam will spread geometrically as in Figure 1. Otherwise there will be one limiting ray which splits at that surface of the isovelocity layer which is closer to the projector, one branch travelling horizontally along this surface, as in Figure29C.

4. When the velocity both above and below the projector is higher than at the level of the projector, there will be a sound channel as described on page 32. At long ranges the whole channel will be filled with rays, but at short ranges there may be "skip distances" or areas inside the channel through which no rays pass. As shown in Figure 29E, a considerable number of rays become limiting rays in some region of the sound channel, and no simple rule exists for predicting which rays they are.

5. Under rather restricted circumstances there may be a strong positive velocity gradient extending over a considerable range of depth below the projector. This may be due to a strong positive temperature gradient, or more commonly to a strong positive density gradient arising when fresh water from a large river flows out over the sea. Under these conditions all the

rays are refracted upward as shown in Figure 29F, and the outer shadow zone can be considered as fused with the inner zone which is usually considered as being bounded by the rays leaving the projector at six degrees above and six degrees below the horizontal. With such strong upward refraction the shadow zone will exist at all ranges below the level at which the lower six degree ray is horizontal. As the portion of the cone of rays outside the six degree rays contains an appreciable amount of energy, there will be a cone of weak intensity represented by, say, the eight, ten, and twelve degree rays, which will be refracted downward below the depth of maximum velocity, and will separate the outer from the inner shadow zone.

E. Construct Ray Diagram.

By use of the Refraction Slide Rule, and Section II of this manual, a ray diagram can be constructed for any bathythermograph slide in a relatively short time. After the possibility of reflections from surface and bottom has been considered, the rule is that targets in the sound beam at ranges less than about 3000-4000 yards will be detected, those in a shadow zone will not. Ray diagrams must be used in cases too complicated to be covered by the "Assured Range" methods, or in cases where more detail is wanted. It should be noted that in many cases a qualitative familiarity with the intensity contours in Figure 29A-29H below may help to interpret a ray diagram in terms of the maximum ranges obtainable.

F. Construct Intensity Diagram.

The primary function of the intensity diagram is to indicate parts of the sound beam in which the signal is relatively weak and an echo is less likely to be detected. Without exact knowledge, this procedure is of course only an approximate one, since the contours do not indicate the intensities of the returning echo, but give simply the ratio of the sound intensity incident on the target to the intensity at a distance of one yard from the projector. Nevertheless this relative information can be very useful; in Figure 28, for instance, it is evident that a submarine below the thermocline at 2500 yards, is less likely to be detected than one above the thermocline at the same range, since the intensities below 100 feet are all considerably reduced.

In the absence of a method for calculating intensities under all conditions, the following rules and sample diagrams will be useful.

1. The intensity drops sharply at the limiting ray in the surface layer when the temperature is constant or increasing with depth unless reflection at the surface occurs, as in Figure 29B and 29C. It drops sharply at the limiting ray at all

depths when the maximum temperature is at the surface as in Figure 29D where it is shown that the intensity at two points close to the limiting ray is large , as compared with the situation in Figure 29B.

2. When a thermocline exists, the intensity contours bend sharply to the left at the top of the thermocline as in Figures 29A, and 29B. Below this level the intensity drops gradually to the limiting ray. Observation has shown that the decrease in intensity below the thermocline at any given range is even larger than the intensity calculations show, indicating that other factors besides refraction are effective.

3. If the BT trace is concave to the right, crossing of rays from different parts of the beam occur as in Figure 29D and 29F. The distribution of intensity will be complex as shown in Figure 29F, where the intensity at the surface reaches a sharp maximum at about 800 yards. Such a maximum may produce a false echo which is distinguishable from a real one only by the fact that it is heard all around the horizon.

4. In shallow water with a bottom of smooth hard sand, reflection of the downward refracted beam may extend the range at the surface significantly as shown in Figure 29G, which also shows that skip distances are to be expected. When the sea surface is calm and smooth, reflection from it may increase ranges near the surface considerably.

5. Observation has shown that the lowest useful intensity for echo-ranging is, on the average -65 to -85db. If the efficiency of the installation has not been tested, the value -75 db may be used. At long ranges this contour rather than the limiting ray must be considered as marking the limiting range.

G. Exact Prediction of Maximum Range.

For a more precise evaluation of the maximum range predicted by the refraction theory, it is necessary to determine which intensity contour marks the limit for the given ship and target under the existing conditions. It is apparent at once that the limit could not be expected to be the same for all targets, all types of sound gear, or all speeds of a given vessel. The best and most obvious procedure is to measure the limiting range on a particular target, observe the water temperatures, calculate the intensity diagram, and note the intensity at which the contact was lost. This is most easily done in deep water where there is no sound reflected from the bottom to confuse the issue, and where there is an isothermal layer to transmit out to the limiting range. An isothermal layer 150 to 200 feet deep is sufficient, and in this case the intensity contour at limiting range can be determined without calculation by inspection of Figure 25.

For example, suppose the limiting range at 15 knots, on a surface target, is 4000 yards, and the isothermal layer is 200 feet deep. Scales 7 and 8 of the slide rule indicate that the sound ray remains in the isothermal layer, and that Figure 26 may be used to get the intensity. The value -72 decibels obtained thus may be taken as the limiting intensity contour for that particular target, and applied to any intensity diagram provided the speed of the boat, the efficiency of the sound gear and the reflecting power of the target may be considered relatively unchanged.

This determination of limiting range in isothermal water may be referred to as "calibration". The practical value of this procedure in anti-submarine warfare is diminished by the fact that calibration is most conveniently carried out on surface targets, rather than on submarines. Not much is known concerning the relative reflecting powers of submarines and surface vessels, although the difference of echo intensity between a submarine and an average freighter is probably not greater than 10 db, and can be determined and allowed for.

In any case, however, too great reliance cannot be placed on the exact predictions of maximum range made on this basis. Factors other than refraction have some effect on sound ranges, and until these factors have been investigated more carefully, detailed predictions cannot be trusted. It is known that refraction is the dominant factor when sound-ranging conditions are bad; i.e. when a shadow zone exists close to the projector, a target inside the shadow zone can usually not be detected with supersonic sound, while a target not in a shadow zone can usually be detected. In other cases, however, the picture is less certain and more information is necessary. Use of the Refraction Slide Rule to analyze the refraction sound field will help to detect discrepancies from the predictions of the refraction theory. Correlation of these discrepancies with oceanographic and other factors should make possible a more precise prediction of maximum ranges in all cases.

Among the factors other than refraction which may be important in their effect on sound-ranging conditions are:

1. The reflecting power of the target.

This is primarily controlled by the area which the target presents to the sonic beam. It is evident that this will vary markedly for different ships and, especially in the case of a submarine, for different orientations of the same ship relative to the sound beam. The reflecting power of the target may also vary with the material of which the target is made.

2. Reverberation.

In shallow water this arises primarily from the bottom and may prove very troublesome. On the other hand, when the bottom is such that the beam is reflected as from a mirror, the maximum range may be extended by the reflected sound (see Example 4, Page 19). Even in deep water, reverberations may occasionally mask a distant echo.

3. Sound absorption by the water.

The absorption and scattering of sound in a wake, an eddy, or a tide rip, for instance, may decrease the range at which a target can be detected. Where the water is relatively undisturbed, sound absorption is usually small, but may occasionally be important, especially at ranges in excess of 4000 yards.

4. Variations in the sound projector and receiver.

If the driving circuit is not accurately tuned to the projector, the power emitted will be seriously affected. Other difficulties with equipment may give rise to electrical "noise" which may mask the echo. The purpose of the calibration described above is to eliminate the effect of all such factors, but if the equipment has changed since the last calibration, deviations between the predicted maximum range and the observed range may be expected.

5. Heavy weather.

When the sea is heavy, the pitching of the ship may produce a stream of bubbles down to and around the sound projector, with a resultant "quenching" of the sound beam and very bad sound conditions. In addition, such conditions raise the noise level, and also cause the projector to point in the wrong direction for much of the time, adding to the difficulties of echo ranging.

It must not be assumed that the above list of additional effects is a complete one. It is possible that there are other effects also which occasionally are important in their effect on echo ranging. There is reason to believe, however, that the above effects include most of those which are commonly encountered. The use of the Refraction Slide Rule will help to determine the extent to which the above factors affect practical echo ranging.

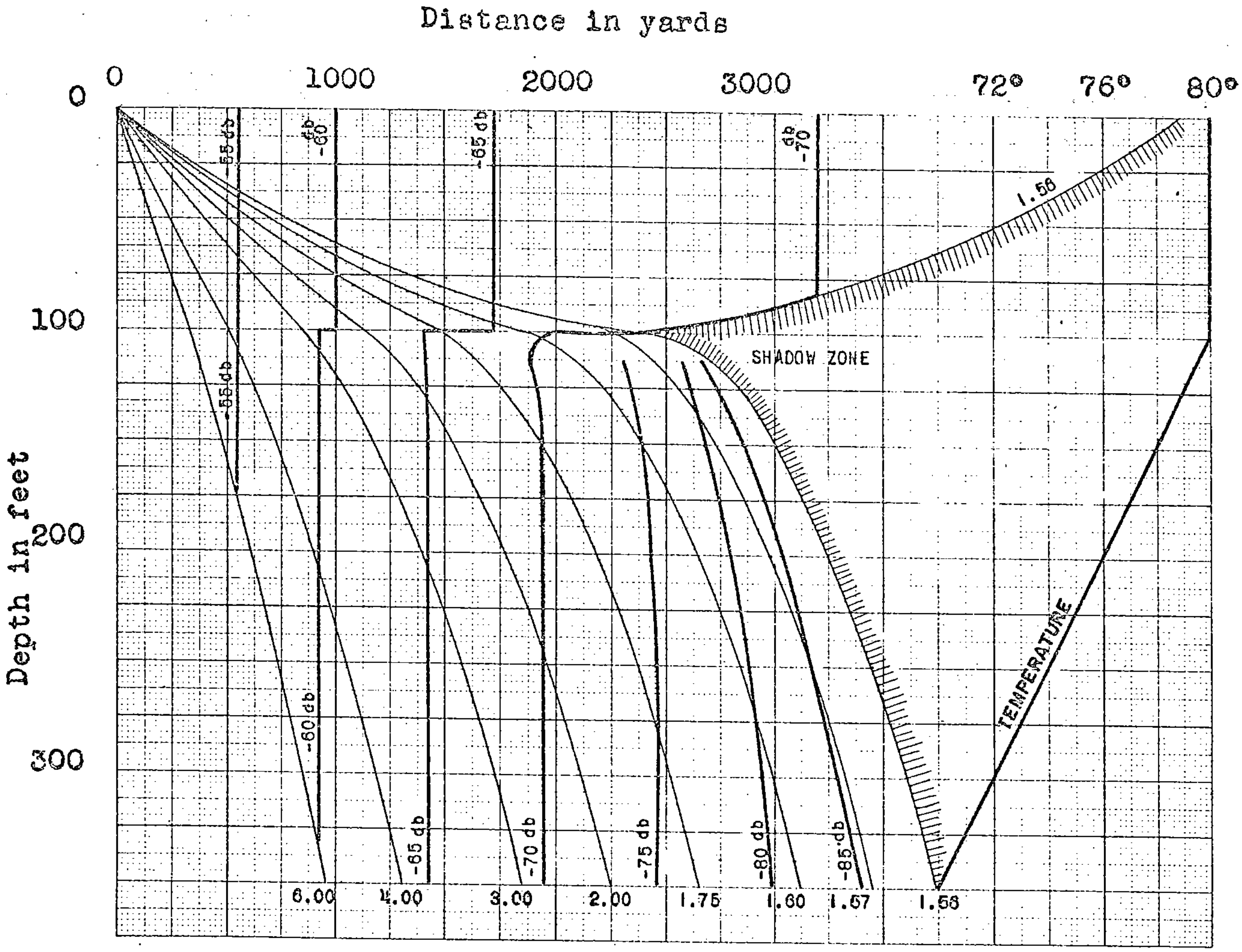


Figure 28 Ray and Intensity Diagram for Examples 7 and 8 and Tables V and VI.

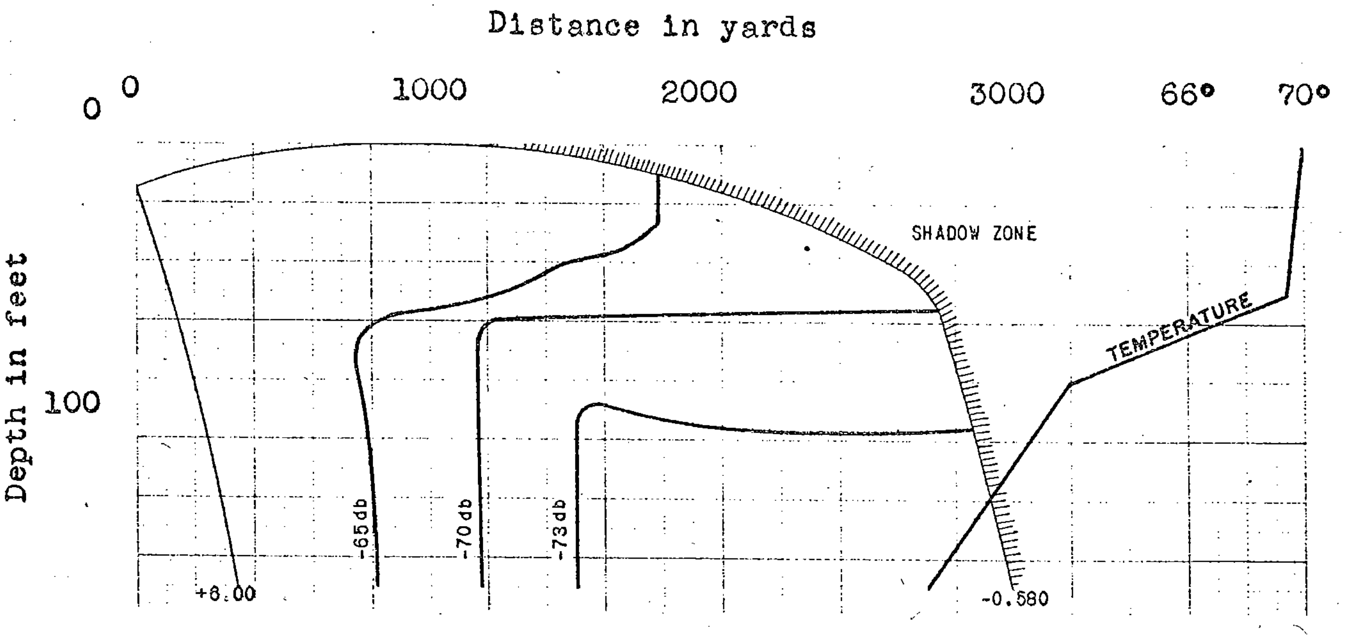


Figure 29A Ray and Intensity Diagram For Water with Negative Thermal Gradient.

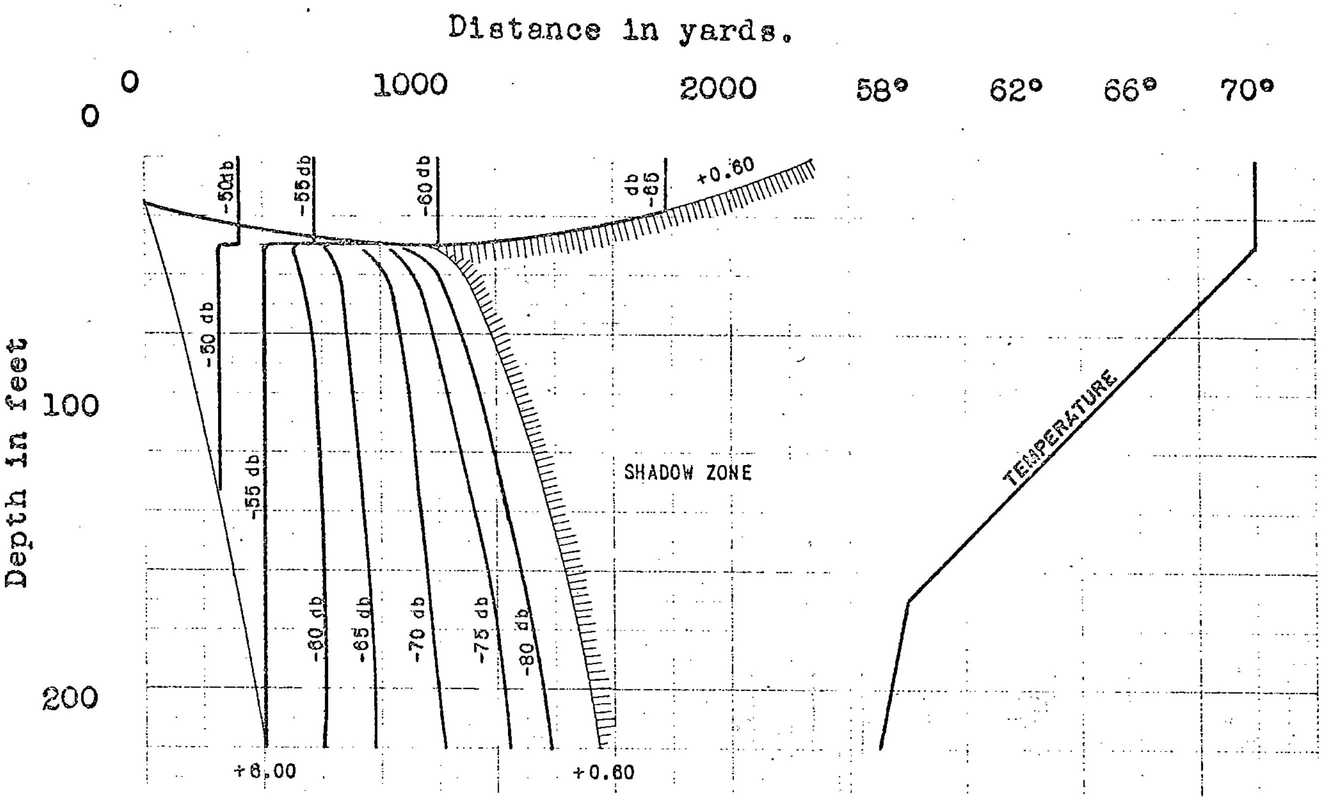


Figure 29B Ray and Intensity Diagram for Water with Isothermal Layer over Negative Thermal Gradient.

For Explanation of these and Following Figures See Pages 38-40.

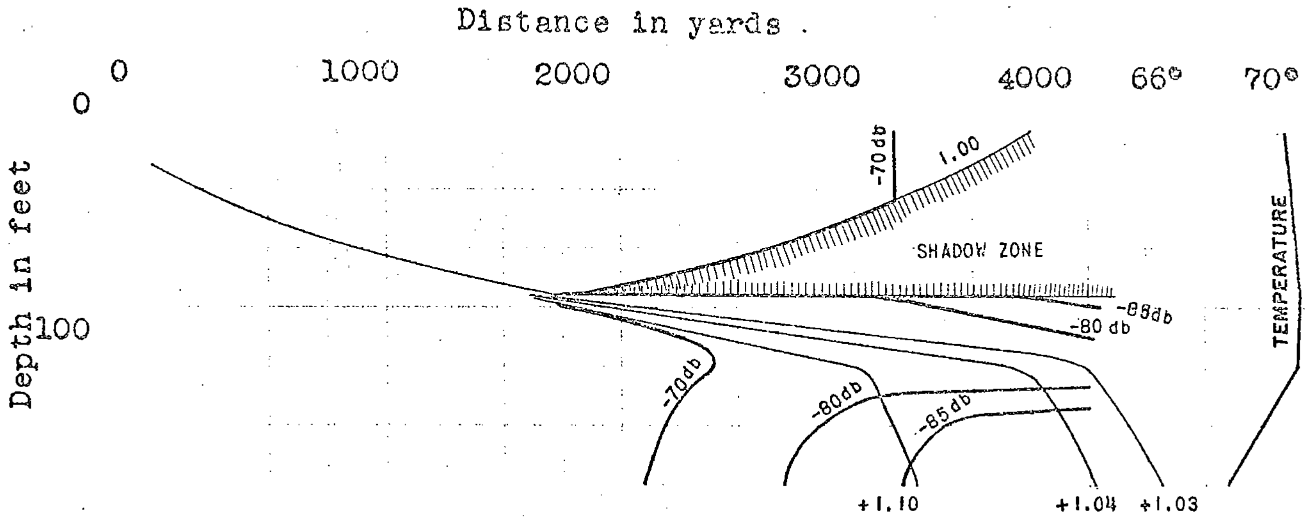


Figure 29C   Ray and Intensity Diagram for Water with Isovelocity Layer.

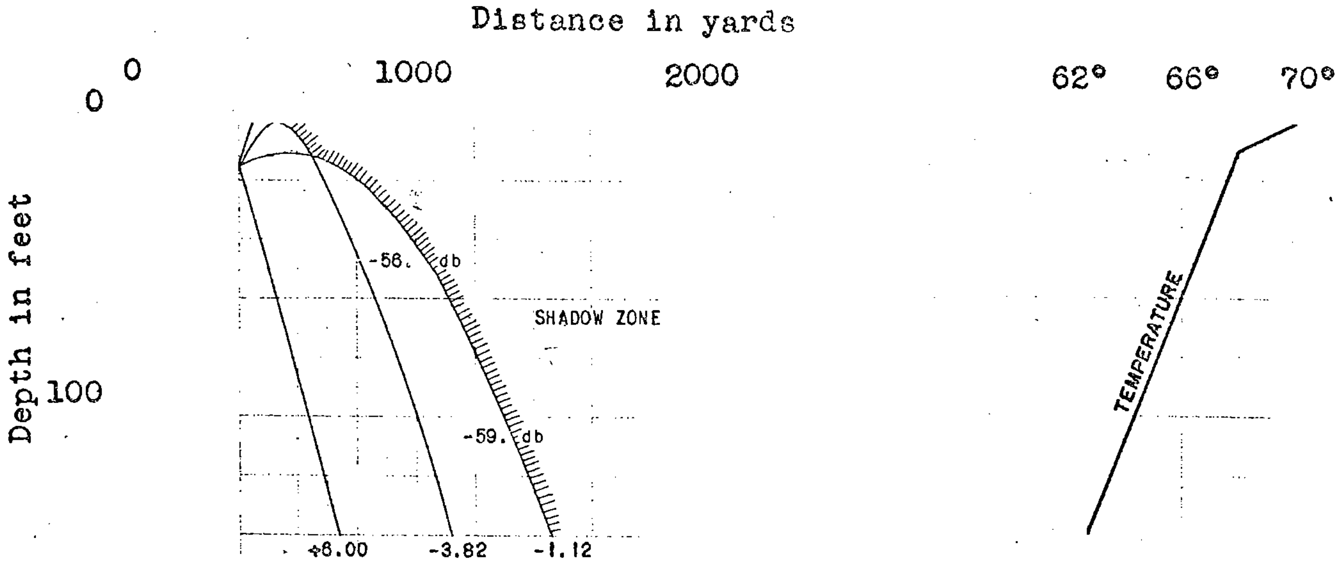


Figure 29D Ray Diagram for Water with Two Negative Thermal Gradients.

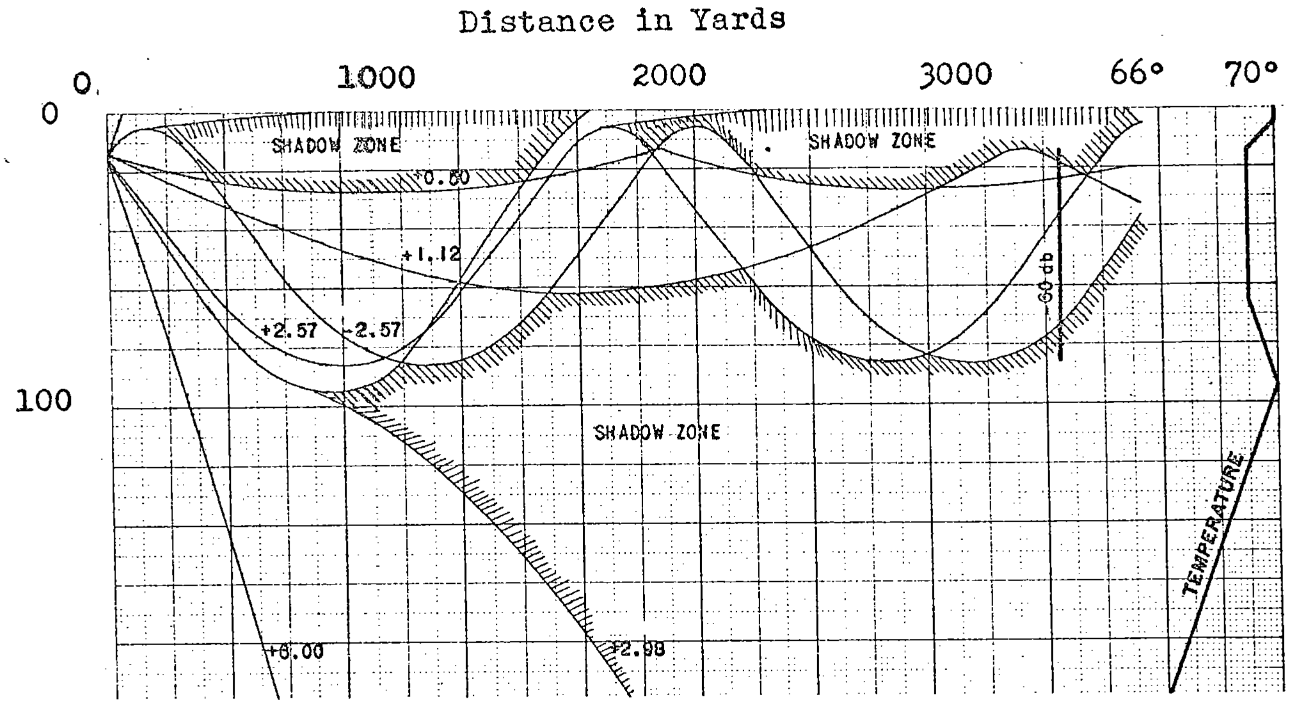
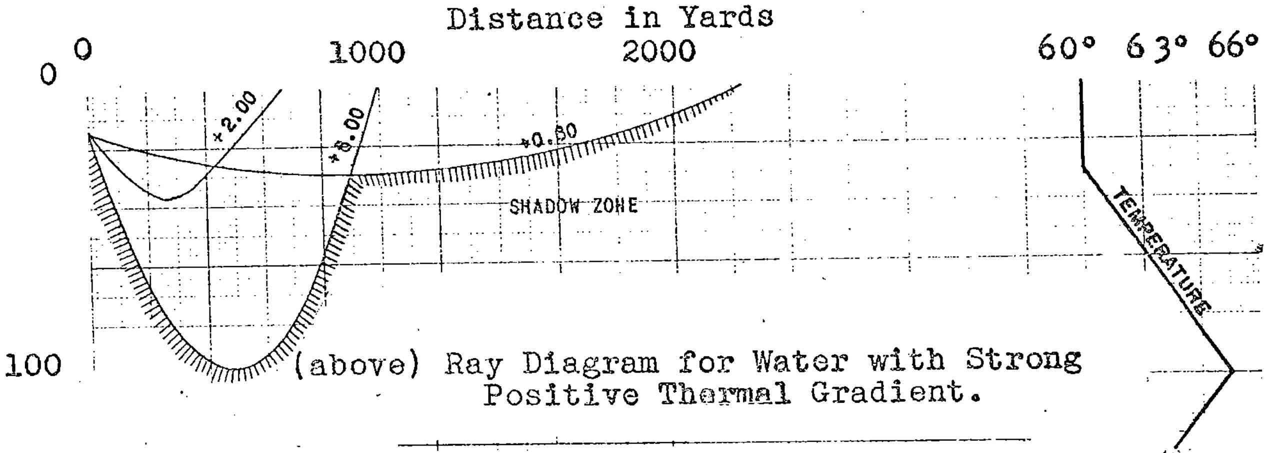
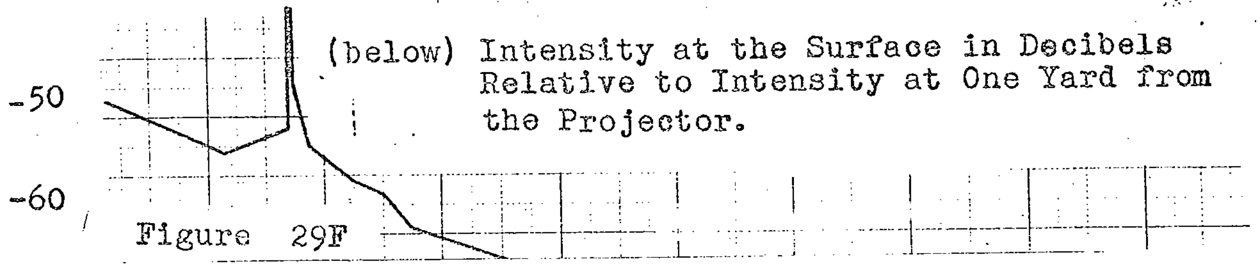


Figure 29E Ray and Intensity Diagram For a Sound Channel.



(above) Ray Diagram for Water with Strong Positive Thermal Gradient.



(below) Intensity at the Surface in Decibels Relative to Intensity at One Yard from the Projector.

Figure 29F

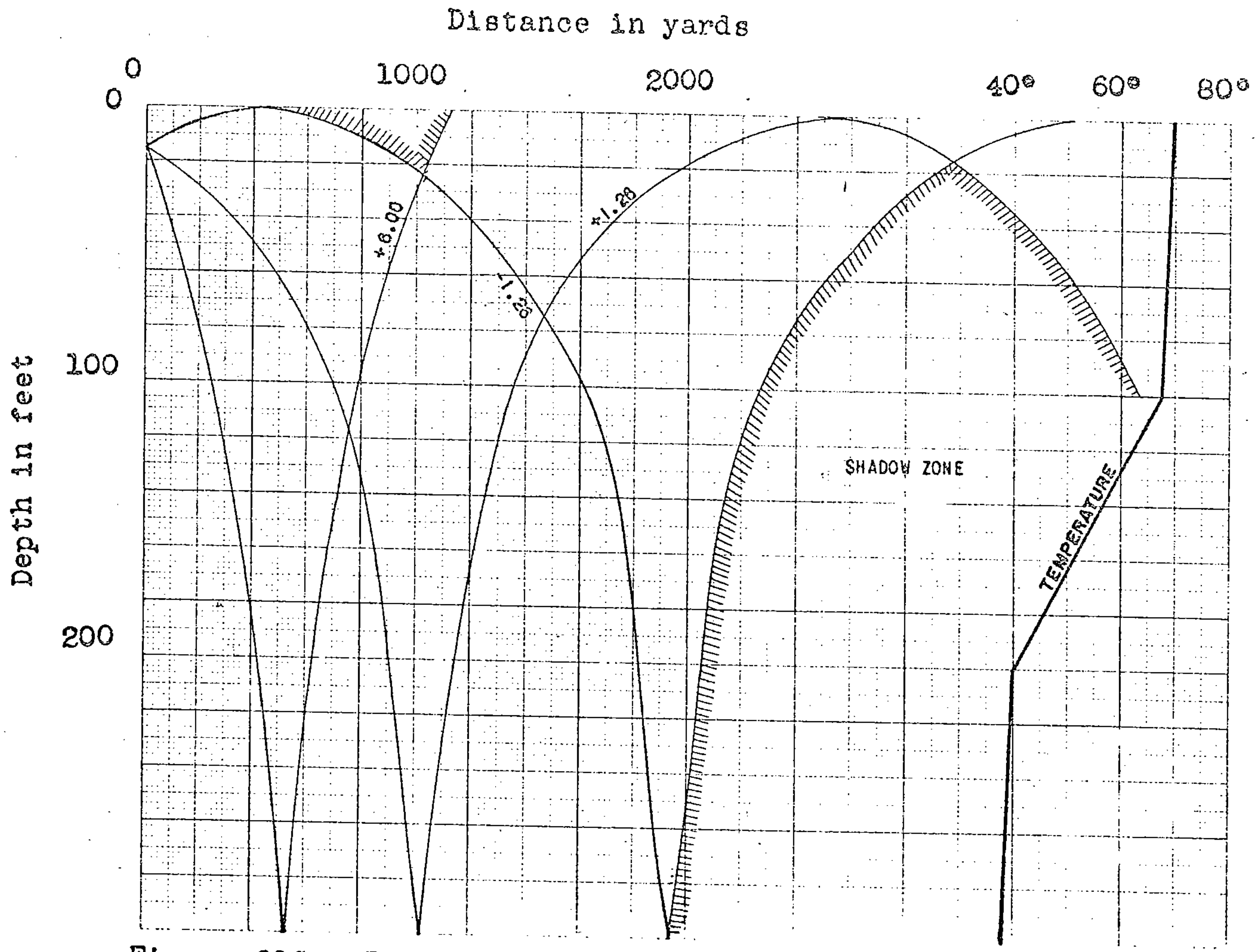


Figure 29G Ray Diagram for Water over a Smooth, Hard Bottom.

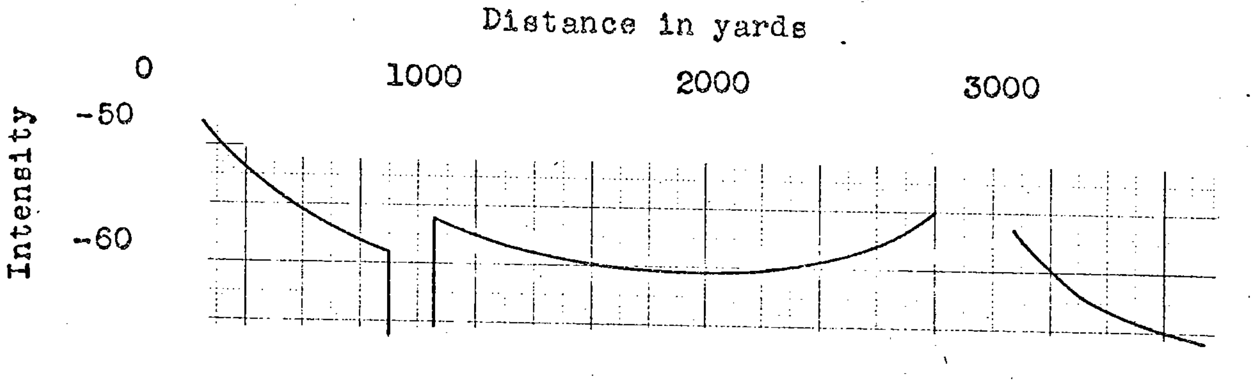


Figure 29H Intensity at Projector Depth in Decibels Relative to Intensity at One Yard From the Projector for Ray Diagram In Figure 29G.

☆U.S. GOVERNMENT PRINTING OFFICE : 1943